The Standard Model: Electroweak Theory & Higgs Physics

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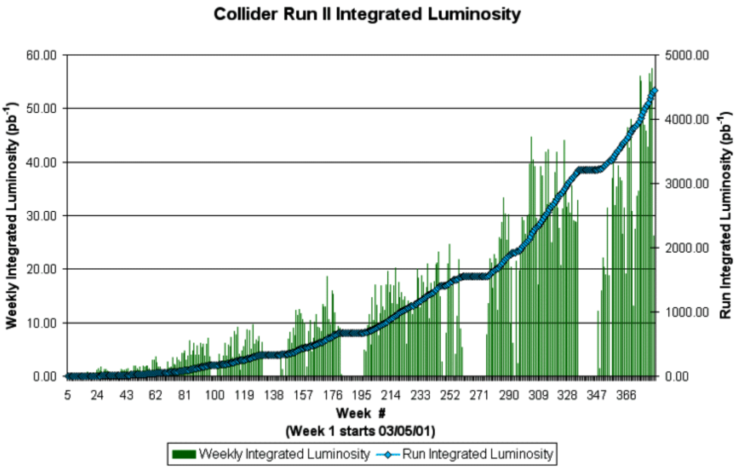
High Energy Physics & Cosmology · Πάφος 2008

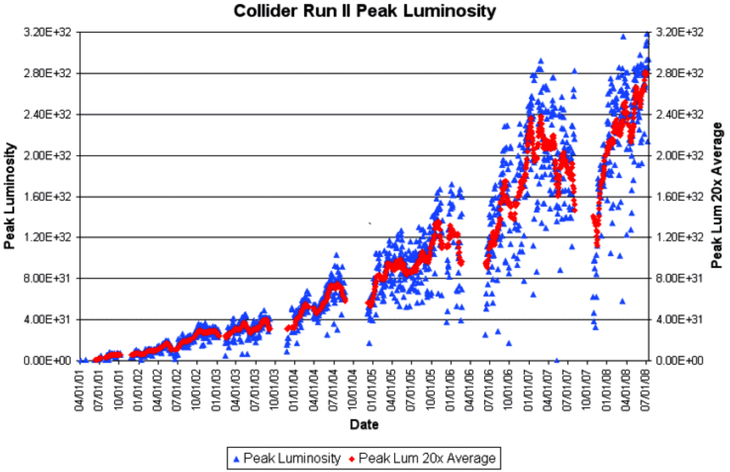
A Decade of Discovery Past . . .

- EW theory $\rightarrow$ law of nature [Z, e^+e^-, $\bar{p}p$, νN, $(g-2)_\mu, \ldots$]
- Higgs-boson influence in the vacuum [EW experiments]
- ν oscillations: $\nu_\mu \rightarrow \nu_\tau$, $\nu_e \rightarrow \nu_\mu/\nu_\tau$ [$\nu_\odot$, ν_{atm}, reactors]
- Understanding QCD [heavy flavor, Z^0, $\bar{p}p$, νN, ep, ions, lattice]
- Discovery of top quark [$\bar{p}p$]
- Direct $\mathcal{CP}$ violation in $K \rightarrow \pi\pi$ [fixed-target]
- B-meson decays violate $\mathcal{CP}$ [$e^+e^- \rightarrow B\bar{B}$]
- Flat universe: dark matter, energy [SN Ia, CMB, LSS]
- Detection of ν_τ interactions [fixed-target]
- Quarks, leptons structureless at 1 TeV scale [mostly colliders]

Tevatron Collider is breaking new ground in sensitivity



Collider Run II Integrated Luminosity
60.00
50.00
40.00
30.00
20.00
10.00
0.00
5000.00
4000.00
3000.00
2000.00
1000.00
0.00
Weekly Integrated Luminosity (pb-1)
Run Integrated Luminosity (pb-1)
5 24 43 62 81 100 119 138 157 176 195 214 233 252 271 290 309 328 347 366
Week #
(Week 1 starts 03/05/01)
Weekly Integrated Luminosity
Run Integrated Luminosity


Collider Run II Peak Luminosity
Peak Luminosity
Peak Lum 20x Average
3.20E+32
2.80E+32
2.40E+32
2.00E+32
1.60E+32
1.20E+32
8.00E+31
4.00E+31
0.00E+00
04/01/01
07/01/01
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01/01/08
04/01/08
07/01/08
Date
▲ Peak Luminosity ◆ Peak Lum 20x Average

Tevatron Collider in a Nutshell

980-GeV protons, antiprotons (2π km)

frequency of revolution $\approx 45\,000\ \mathrm{s}^{-1}$

392 ns between crossings

($36 \otimes 36$ bunches)

collision rate $= \mathcal{L} \cdot \sigma_{\text{inelastic}} \approx 10^7\ \mathrm{s}^{-1}$

$c \approx 10^9$ km/h; $\quad v_p \approx c - 495$ km/h

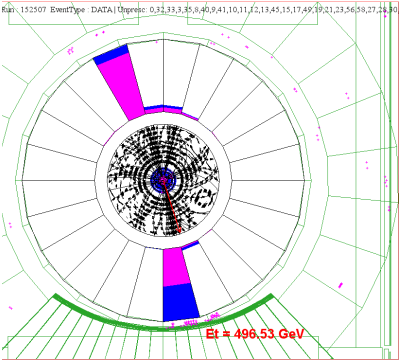
Record $\mathcal{L}_{\text{init}} = 3.1849 \times 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

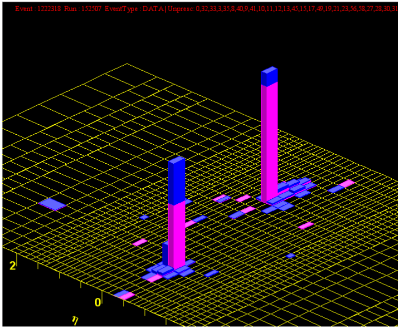
[CERN ISR: pp, $1.4 \times 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$]

Goal: $\approx 8\ \mathrm{fb}^{-1}$ by 10.2009

The World's Most Powerful Microscopes

nanonanophysics

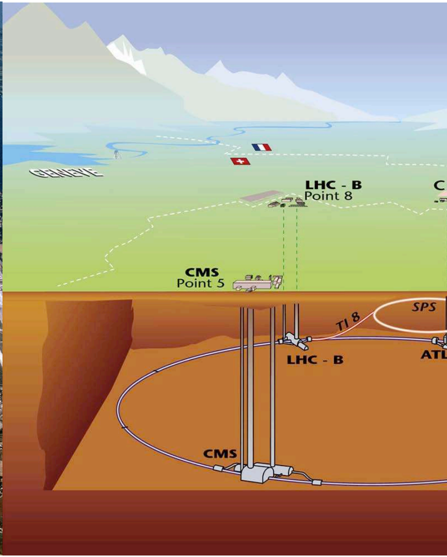




CDF dijet event ($\sqrt{s} = 1.96$ TeV): $E_T = 1.364$ TeV

$$\boxed{q\bar{q} \longrightarrow \text{jet} + \text{jet}}$$

LHC will operate *soon*, breaking new ground in E & $\mathcal{L}$



LHC in a nutshell

7-TeV protons on protons (27 km); $v_p \approx c - 10$ km/h

Heavy ions, e.g. Pb-Pb at $\sqrt{s} \approx 1$ PeV

Novel two-in-one dipoles (≈ 9 teslas)

Beam commissioning soon

$\leadsto$ First collisions at $E_{cm} = 10$ TeV

Pilot run

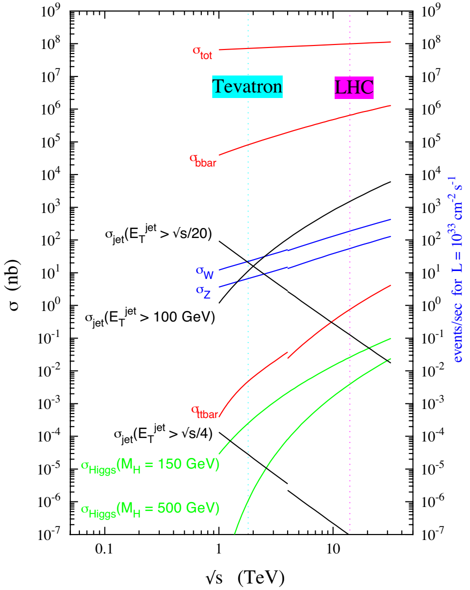
First physics run in 2009

Eventual: $\mathcal{L} \gtrsim 10^{34}$ cm^{-2} s^{-1}: 100 fb^{-1}/year

Why the LHC is so exciting (I)

- Thorough exploration of 1-TeV scale
- Even low luminosity opens vast new realm:
  10 pb^{-1} *(few days at initial $\mathcal{L}$)* yields
  8000 top quarks, 10^5 W-bosons,
  100 QCD dijets beyond Tevatron kinematic limit
  Supersymmetry hints in a few weeks ?
- Essential first step: rediscover the standard model
- The antithesis of a one-experiment machine;
  enormous scope and versatility beyond high-$p_\perp$

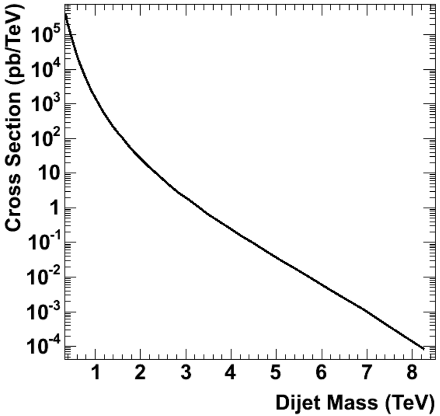
Sample event rates in $p^{\pm}p$ collisions

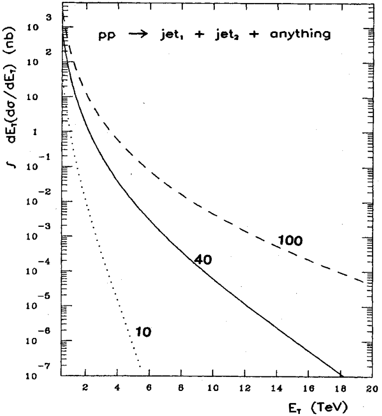


LHC experiments will need a trigger . . .

$d\sigma/d\mathcal{M}_{jj}$, $|\eta| \leq 1$

$\int dE_T d\sigma/dE_T$, $|y_{1,2}| \leq 2.5$

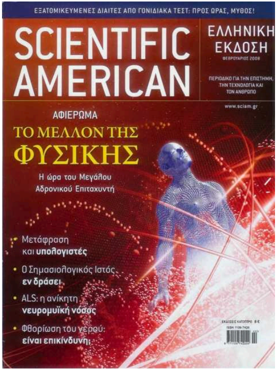




Dijet integral cross section, $|\eta| \leq 2.5$ . . .

$10\ \mathrm{pb}^{-1}$ @ LHC $\rightsquigarrow \gtrsim 10^4$ events with $E_T \gtrsim 1.364$ TeV

The Future of Physics / ΤΟ ΜΕΛΛΟΝ ΤΗΣ ΦΥΣΙΚΗΣ



"The Coming Revolutions in Particle Physics," *Scientific American* February 2008.

'Η ΕΠΕΡΧΟΜΕΝΗ ΕΠΑΝΑΣΤΑΣΗ ΣΤΗ ΣΟΜΑΤΙΔΙΑΚΗ ΦΥΣΙΚΕ,' *Scientific American* ΕΛΛΗΝΙΚΗ ΕΚΔΟΣΗ, ΦΕΒΡΟΥΑΡΙΟΣ 2008

Our picture of matter

Pointlike constituents ($r < 10^{-18}$ m)

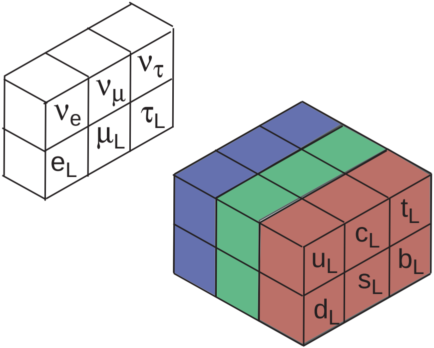
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

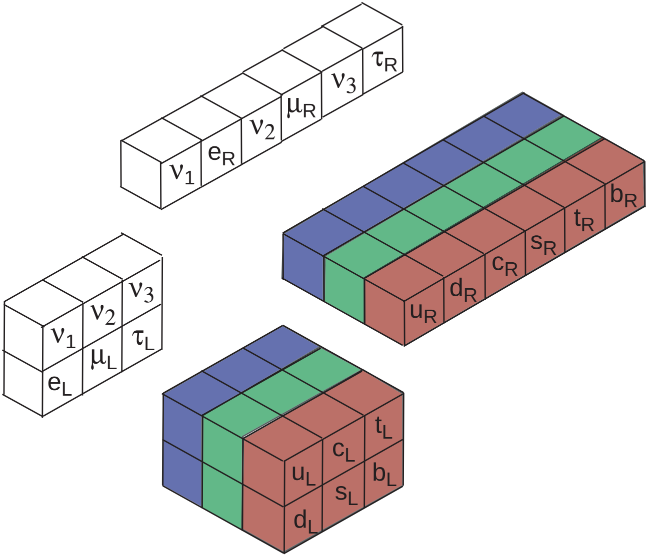
Few fundamental forces, derived from gauge symmetries

$$\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$$

Electroweak symmetry breaking: Higgs mechanism?



ν_τ
ν_μ
ν_e
τ_L
μ_L
e_L
t_L
c_L
u_L
b_L
s_L
d_L



ν_1
e_R
ν_2
μ_R
ν_3
τ_R
ν_1
ν_2
ν_3
e_L
μ_L
τ_L
u_R
d_R
c_R
s_R
t_R
b_R
u_L
d_L
c_L
s_L
t_L
b_L

Parity violation in weak decays

1956 Wu *et al.*: correlation between spin vector $\vec{J}$ of polarized ^{60}Co and direction $\hat{p}_e$ of outgoing β particle

Parity leaves spin (axial vector) unchanged $\boxed{\mathcal{P} : \vec{J} \to \vec{J}}$

Parity reverses electron direction $\boxed{\mathcal{P} : \hat{p}_e \to -\hat{p}_e}$

Correlation $\vec{J} \cdot \hat{p}_e$ is *parity violating*

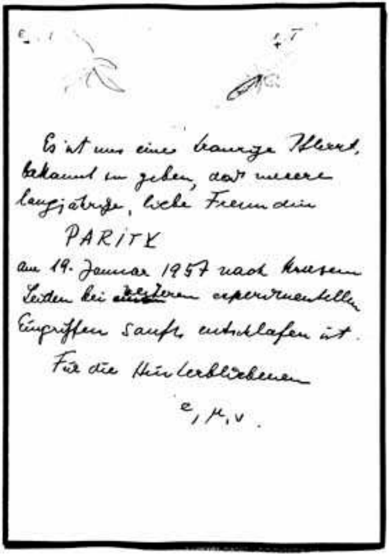
Late 1950s: (charged-current) weak interactions are left-handed

Parity links left-handed, right-handed ν,

$$\nu_L \xrightarrow{\Lleftarrow} \mathcal{P} \xleftarrow{\Lleftarrow} \cancel{\nu}_R$$

$\Rightarrow$ build a manifestly parity-violating theory with only ν_L.

Pauli's Reaction to the Downfall of Parity



Es ist uns eine traurige Pflicht, bekannt zu geben, daß unsere langjährige, liebe Freundin

PARITY

am 19. Januar 1957 nach kurzem Leiden bei weiteren experimentellen Eingriffen sanft entschlafen ist.

Für die Hinterbliebenen

e, μ, ν.

Pauli's Reaction to the Downfall of Parity

Es ist uns eine traurige Pflicht, bekannt zu geben, daß unsere langjährige ewige Freundin

PARITY

den 19. Januar 1957 nach kurzen Leiden bei weiteren experimentellen Eingriffen sanfte entschlafen ist.

Für die hinterbliebenen

$e \quad \mu \quad \nu$

It is our sad duty to announce that our loyal friend of many years

PARITY

went peacefully to her eternal rest on the nineteenth of January 1957, after a short period of suffering in the face of further experimental interventions.

For those who survive her,

$e \quad \mu \quad \nu$

How do we know ν is left-handed?

▷ $\boxed{\nu_\mu}$ Measure μ^+ helicity in (spin-zero) $\pi^+ \to \mu^+\nu_\mu$

$$\nu_\mu \xleftarrow{\Rightarrow} \pi^+ \xrightarrow{\Leftarrow} \mu^+$$

$\boxed{h(\nu_\mu) = h(\mu^+)}$ Bardon, PRL **7,** 23 (1961); Possoz, PL **70B,** 265 (1977)

μ^+ forced to have "wrong" helicity

...inhibits decay, and inhibits $\pi^+ \to e^+\nu_e$ more

$$\Gamma(\pi^+ \to e^+\nu_e)/\Gamma(\pi^+ \to \mu^+\nu_\mu) = 1.23 \times 10^{-4}$$

▷ Longitudinal pol. of recoil nucleus in $\mu^{-12}\mathrm{C}(J=0) \to {}^{12}\mathrm{B}(J=1)\nu_\mu$

$\boxed{\text{Infer } h(\nu_\mu) \text{ by angular momentum conservation}}$

Roesch, *Am. J. Phys.* **50**, 931 (1981)

▷ $\boxed{\nu_e}$ Measure longitudinal polarization of recoil nucleus in

$$e^- \, {}^{152}\mathrm{Eu^m}(J=0) \;\to\; {}^{152}\mathrm{Sm}^*(J=1)\,\nu_e$$
$$\hookrightarrow \gamma \, {}^{152}\mathrm{Sm}$$

$\boxed{\text{Infer } h(\nu_e) \text{ from } \gamma \text{ polarization}}$

Goldhaber, *Phys. Rev.* **109**, 1015 (1958)

▷ $\boxed{\nu_\tau}$ Variety of determinations in $\tau \to \pi\nu_\tau$, $\tau \to \rho\nu_\tau$, etc.

e.g., Abe, *et al.* (SLD), *Phys. Rev. Lett.* **78**, 4691 (1997)

Charge conjugation is also violated . . .

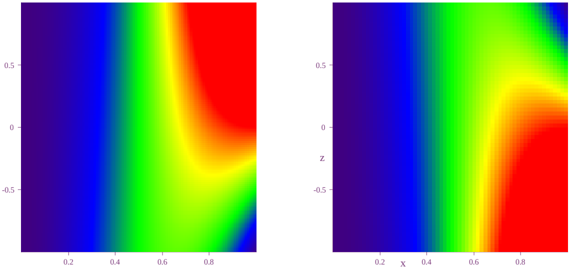
$$\nu_L \xrightarrow{\Leftarrow} \ \mathcal{C} \ \xrightarrow{\Leftarrow} \cancel{\bar{\nu}_L}$$

$\mu^{\pm}$ decay: angular distributions of $e^{\pm}$ reversed

$$\frac{dN(\mu^{\pm} \to e^{\pm} + \ldots)}{dxdz} = x^2(3-2x)\left[1 \pm z\frac{(2x-1)}{(3-2x)}\right]$$

$$x \equiv p_e/p_e^{\max},\ z \equiv \hat{s}_\mu \cdot \hat{p}_e$$

e^+ follows μ^+ spin e^- avoids μ^- spin



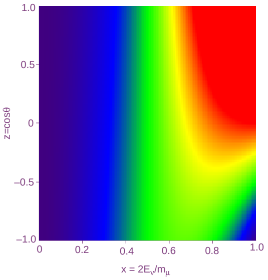
Consequences for neutrino factory

$$\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$$

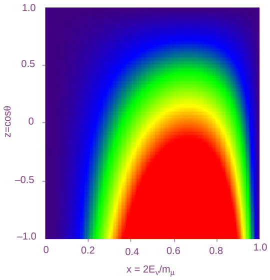
$$\frac{d^2 N_{\bar{\nu}_\mu}}{dxdz} = x^2[(3-2x) - (1-2x)z]\,, \quad x \equiv p_\nu / p_\nu^{\max},\; z \equiv \hat{p}_\nu \cdot \hat{s}_\mu$$

$$\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$$

$$\frac{d^2 N_{\nu_e}}{dxdz} = 6x^2[(1-x)(1-z)]$$



$\bar{\nu}_\mu$



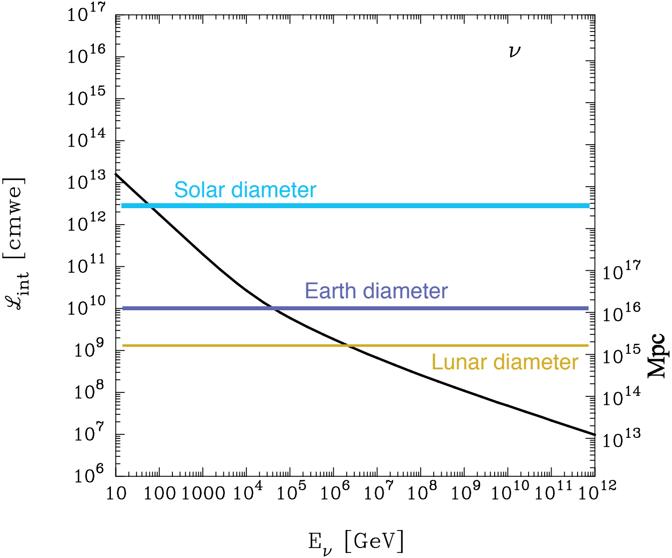
ν_e

Neutrinos are abundant!

- Each second, some 10^{14} neutrinos made in the Sun pass through your body.
- Each second, about 10^{3} neutrinos made in Earth's atmosphere by cosmic rays pass through your body.
- Other neutrinos reach us from natural (radioactive decays of elements inside the Earth) and artificial (nuclear reactors, particle accelerators) sources.
- Inside your body are more than 10^{7} neutrino relics from the early universe.

ν_i, $\bar{\nu}_i$ number density now: 56 cm^{-3}, $\propto (1+z)^3$

$\nu N \to \mu + \ldots$ Interaction Lengths



Don't be neutrinos!

Elementarity

▷ Are quarks and leptons structureless?

Symmetry

▷ Electroweak symmetry breaking and the 1-TeV scale

▷ Origin of gauge symmetries

▷ Meaning of discrete symmetries

Unity

▷ Coupling constant unification

▷ Unification of quarks and leptons: (neutrality of atoms $\Rightarrow$ new forces!)
of constituents and force particles

▷ Incorporation of gravity

Identity

▷ Fermion masses and mixings; CP violation; ν oscillations

▷ What makes an electron an e and a top quark a t?

Topography

▷ What is the fabric of space and time? . . . the origin of space and time?

Two views of Symmetry

1. *Indistinguishability*
   One object transformed into another

   Familiar (and useful!) from

   Global Symmetries: isospin, $SU(3)_f$, ...
   Spacetime Symmetries
   Gauge Symmetries

“EQUIVALENCE”

Idealize more perfect worlds, the better to understand our diverse, changing world

Unbroken unified theory: perfect world of equivalent forces, interchangeable massless particles ... Perfectly boring?

Symmetry $\Leftrightarrow$ *Disorder*

The Perfect World

Two views of Symmetry

2. *Unobservable*

   Goodness of a symmetry means something cannot be measured

   e.g., vanishing asymmetry

Unobservable	Transformation	Conserved
Absolute position	$\vec{r} \to \vec{r} + \vec{\Delta}$	$\vec{p}$
Absolute time	$t \to t + \delta$	E
Absolute orientation	$\hat{r} \to \hat{r}'$	$\vec{L}$
Absolute velocity	$\vec{v} \to \vec{v} + \vec{w}$	
Absolute right	$\vec{r} \to -\vec{r}$	P
Absolute future	$t \to -t$	T
Absolute charge	$Q \to -Q$	C
Absolute phase		
$\vdots$		

Unity

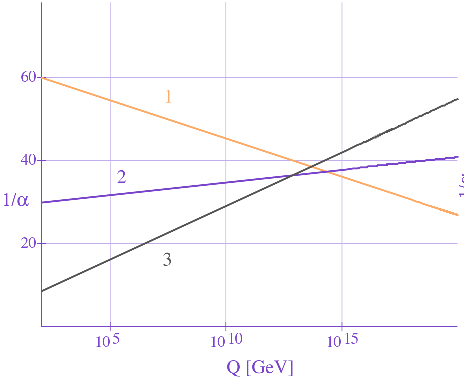
Unification $\approx$ Generalization is the essence of scientific explanation.

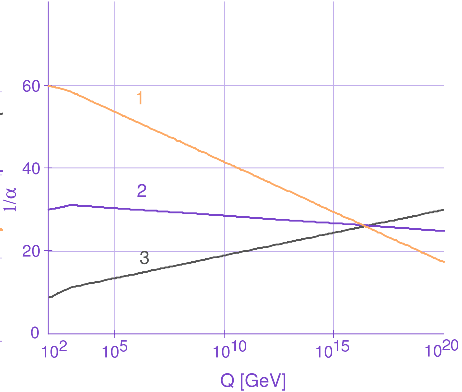
Examples:

- Electricity + magnetism + light (Maxwell)
- Atomic theory: relates thermodynamics and statistical mechanics to Newtonian mechanics
- Chemistry + electricity (Faraday)
- Atomic structure + chemistry + quantum mechanics

Unity

Coupling-constant unification?





The Idea of Gauge Invariance

Maxwell's equation for magnetic charge,

$$\nabla \cdot \mathbf{B} = 0, \tag{1}$$

invites us to write the magnetic field as

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{2}$$

where **A** is called the vector potential. This identification ensures that **B** will be divergenceless, by virtue of the identity

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0. \tag{3}$$

If we add an arbitrary gradient to the vector potential

$$\mathbf{A} \rightarrow \mathbf{A} + \nabla\Lambda, \tag{4}$$

the magnetic field is unchanged, because

$$\mathbf{B} = \nabla \times (\mathbf{A} + \nabla\Lambda) = \nabla \times \mathbf{A}. \tag{5}$$

The Idea of Gauge Invariance . . .

The curl equation (Faraday–Lenz) for the electric field,

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t, \tag{6}$$

which can be rewritten as

$$\nabla \times (\mathbf{E} + \partial \mathbf{A}/\partial t) = 0, \tag{7}$$

suggests the identification

$$\mathbf{E} + \partial \mathbf{A}/\partial t = -\nabla V, \tag{8}$$

where V is known as the scalar potential. In order that the electric field remain invariant under the shift (4), we must also require

$$V \to V - \partial \Lambda/\partial t. \tag{9}$$

The Idea of Gauge Invariance . . .

Covariant notation: EM field-strength tensor

$$F^{\mu\nu} = -F^{\nu\mu} = \partial^\nu A^\mu - \partial^\mu A^\nu = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix}, \quad (10)$$

built up from the four-vector potential

$$A^\mu = (V; \mathbf{A}), \quad (11)$$

is unchanged by the "gauge transformation"

$$A^\mu \to A^\mu - \partial^\mu \Lambda, \quad (12)$$

where $\Lambda(x)$ is an arbitrary function of the coordinate. The fact that many different four-vector potentials yield the same electromagnetic fields, and thus describe the same physics, is a manifestation of the gauge invariance of classical electrodynamics.

The Idea of Gauge Invariance . . .

The remaining Maxwell equations,

$$\nabla \cdot \mathbf{E} = \rho = -\nabla \cdot \dot{\mathbf{A}} - \nabla^2 V \ , \tag{13}$$

$$\begin{array}{rcl} \nabla \times \mathbf{B} & = & \mathbf{J} + \dot{\mathbf{E}} = \mathbf{J} - \ddot{\mathbf{A}} - \nabla \dot{V} \\ \| & & \\ \nabla \times (\nabla \times \mathbf{A}) & = & -\nabla^2 \mathbf{A} + \nabla(\nabla \cdot \mathbf{A}), \end{array} \tag{14}$$

correspond, in covariant notation, to $\partial_\mu F^{\mu\nu} = -J^\nu$, with $J^\nu = (\rho; \mathbf{J})$.
EM current is conserved: $\partial_\nu J^\nu = -\partial_\nu \partial_\mu F^{\mu\nu} = 0$.
Wave equation may be expanded as

$$\Box A^\nu - \partial^\nu(\partial_\mu A^\mu) = J^\nu, \tag{15}$$

No sources, Lorenz gauge ($\partial_\mu A^\mu = 0$) $\quad \rightsquigarrow \quad \Box A^\nu = 0$.
Each component of A^μ (photon field) satisfies a Klein–Gordon equation for a massless particle.

Symmetries $\Longrightarrow$ interactions: Phase Invariance in QM

QM state: complex Schrödinger wave function $\psi(x)$

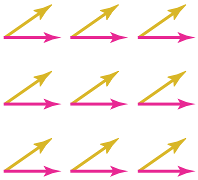
Observables $\langle O \rangle = \int d^n x \psi^* O \psi$ are *unchanged*

under a global phase rotation

$$\psi(x) \rightarrow e^{i\theta}\psi(x)$$
$$\psi^*(x) \rightarrow e^{-i\theta}\psi^*(x)$$

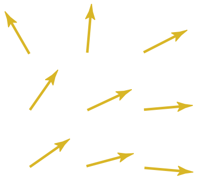
- Absolute phase of the wave function cannot be measured (is a matter of convention).
- Relative phases (interference experiments) are unaffected by a global phase rotation.

Global rotation — same everywhere



Might we choose one phase convention in Πάφος and another in Batavia?

A different convention at each point?



$$\boxed{\psi(x) \to e^{iq\alpha(x)}\psi(x)}$$

There is a price . . .

Some variables (e.g., momentum) and the Schrödinger equation itself contain derivatives. Under the transformation $\psi(x) \to e^{iq\alpha(x)}\psi(x)$, the gradient of the wave function transforms as

$$\nabla\psi(x) \to e^{iq\alpha(x)}[\nabla\psi(x) + iq(\nabla\alpha(x))\psi(x)].$$

The $\nabla\alpha(x)$ term spoils local phase invariance.

To restore local phase invariance, modify eqns. of motion, observables.

$\boxed{\text{Replace } \nabla \text{ by } \nabla + iq\vec{A}}$ "Gauge-covariant derivative"

If the vector potential $\vec{A}$ transforms under local phase rotations as

$$\vec{A}(x) \to \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x),$$

then $(\nabla + iq\vec{A})\psi \to e^{iq\alpha(x)}(\nabla + iq\vec{A})\psi$ and $\psi^*(\nabla + iq\vec{A})\psi$ is invariant under local rotations.

Note ...

- $\vec{A}(x) \to \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x)$ has the form of a gauge transformation in electrodynamics.
- Replacement $\nabla \to (\nabla + iq\vec{A})$ corresponds to $\vec{p} \to \vec{p} - q\vec{A}$

Form of interaction deduced from local phase invariance

Maxwell's equations derived from a symmetry principle

QED is the gauge theory based on $U(1)$ phase symmetry

General procedure . . . also in field theory

- Recognize a symmetry of Nature.
- Build it into the laws of physics.
  (Connection with conservation laws)
- Symmetry in stricter (local) form $\rightsquigarrow$ interactions.

Results in . . .

- Massless vector fields (gauge fields).
- Minimal coupling to the conserved current.
- Interactions among gauge fields, if non-Abelian.

Posed as a problem in mathematics, construction of a gauge theory is always possible (at the level of a classical $\mathcal{L}$; consistent quantum theory may require additional vigilance). Formalism is no guarantee that the gauge symmetry was chosen wisely.

Phase invariance in field theory

Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$$

for a free fermion follows from the Lagrangian

$$\mathcal{L} = \bar{\psi}(x)(i\gamma^\mu \partial_\mu - m)\psi(x),$$

where $\bar{\psi}(x) = \psi^\dagger(x)\gamma^0$, on applying Euler–Lagrange equations,

$$\frac{\partial \mathcal{L}}{\partial \phi(x)} = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi(x))}.$$

Impose local phase invariance:

$$\begin{aligned}
\mathcal{L} &= \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi \\
&= \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - qA_\mu \bar{\psi}\gamma^\mu \psi \\
&= \mathcal{L}_{\text{free}} - J^\mu A_\mu,
\end{aligned}$$

where $J^\mu = q\bar{\psi}\gamma^\mu\psi$ (follows from global phase invariance)

Problem 1

Verify that the Lagrangian $\mathcal{L} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi$ is invariant under the combined transformations $\psi(x) \to e^{iq\alpha(x)}\psi(x)$, $A_\mu(x) \to A_\mu(x) - \partial_\mu \alpha(x)$.

Toward QED

Add kinetic energy term for the vector field, to describe the propagation of free photons.

$$\mathcal{L}_\gamma = -\tfrac{1}{4}(\partial_\nu A_\mu - \partial_\mu A_\nu)(\partial^\nu A^\mu - \partial^\mu A^\nu).$$

Assembling the pieces:

$$\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{free}} - J^\mu A_\mu - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

A photon mass term would have the form

$$\mathcal{L}_\gamma = \tfrac{1}{2}M_\gamma^2 A^\mu A_\mu,$$

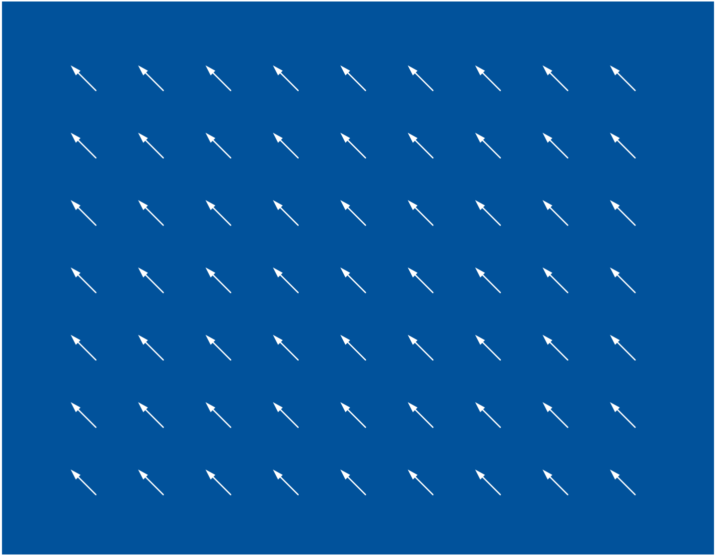
which obviously violates local gauge invariance because

$$A^\mu A_\mu \to (A^\mu - \partial^\mu \alpha)(A_\mu - \partial_\mu \alpha) \neq A^\mu A_\mu.$$

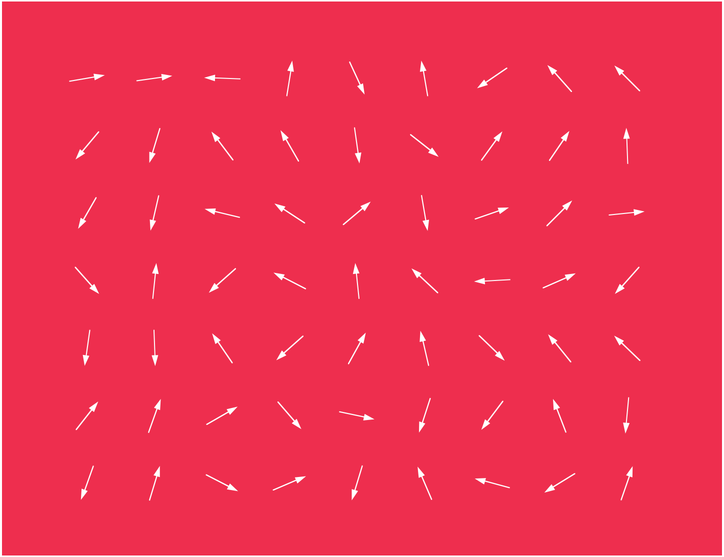
Local gauge invariance $\rightsquigarrow$ massless photon: observe $M_\gamma < 5 \times 10^{-17}$ eV/c^2

Symmetry of laws $\not\Rightarrow$ symmetry of outcomes

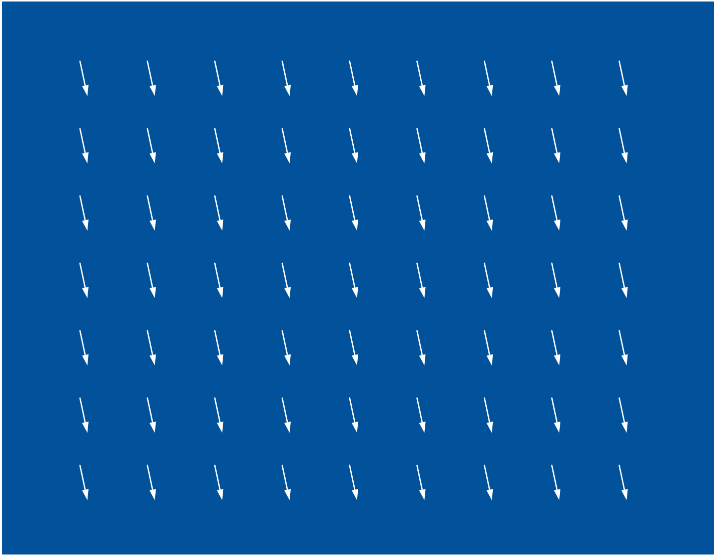
The Crystal World



The Crystal World



The Crystal World



Non-Abelian Gauge Theories

Free-nucleon Lagrangian

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi.$$

written in terms of the composite fermion fields

$$\psi \equiv \begin{pmatrix} p \\ n \end{pmatrix}.$$

Invariant under global isospin rotations $\psi \to \exp(i\boldsymbol{\tau} \cdot \boldsymbol{\alpha}/2)\psi$, and the isospin current $\mathbf{J}^\mu = \bar{\psi}\gamma^\mu \frac{\boldsymbol{\tau}}{2}\psi$ is conserved.

In the absence of EM, complete freedom in naming the proton and neutron; once freely chosen, the convention must be respected everywhere throughout spacetime.

Local isospin invariance?

Non-Abelian Gauge Theories . . .

If under a local gauge transformation the field transforms as

$$\psi(x) \to \psi'(x) = G(x)\psi(x),$$

with

$$G(x) \equiv \exp\left(i\boldsymbol{\tau} \cdot \boldsymbol{\alpha}(x)/2\right),$$

then the gradient transforms as

$$\partial_\mu \psi \to G(\partial_\mu \psi) + (\partial_\mu G)\psi.$$

Introduce a gauge-covariant derivative

$$\mathcal{D}_\mu \equiv I\partial_\mu + igB_\mu,$$

where

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Non-Abelian Gauge Theories . . .

2×2 matrix defined by

$$B_\mu = \tfrac{1}{2}\boldsymbol{\tau} \cdot \mathbf{b}_\mu = \tfrac{1}{2}\tau^a b^a_\mu = \tfrac{1}{2}\begin{pmatrix} b^3_\mu & b^1_\mu - ib^2_\mu \\ b^1_\mu + ib^2_\mu & -b^3_\mu \end{pmatrix}$$

where the three gauge fields are $\mathbf{b}_\mu = (b^1_\mu, b^2_\mu, b^3_\mu)$, bold-face quantities denote isovectors, and the isospin index a runs from 1 to 3.

Require $\mathcal{D}_\mu\psi \to \mathcal{D}'_\mu\psi' = G(\mathcal{D}_\mu\psi)$ to learn how B_μ must behave under gauge transformations. Explicit computation:

$$b'^{\,l}_\mu = b^l_\mu - \varepsilon_{jkl}\alpha^j b^k - \frac{1}{g}\partial_\mu\alpha^l,$$
$$\mathbf{b}'_\mu = \mathbf{b}_\mu - \boldsymbol{\alpha} \times \mathbf{b}_\mu - \frac{1}{g}\partial_\mu\boldsymbol{\alpha},$$

Transformation rule depends on the structure constants ε_{jkl} and not on the representation of the isospin group.

Kinetic term for gauge bosons

So far, we have

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi \\ &= \mathcal{L}_0 - g\bar{\psi}\gamma^\mu B_\mu \psi \\ &= \mathcal{L}_0 - \frac{g}{2}\mathbf{b}_\mu \cdot \bar{\psi}\gamma^\mu \boldsymbol{\tau}\psi,\end{aligned}$$

free Dirac Lagrangian plus interaction coupling isovector gauge fields to conserved isospin current. (Analogous to QED)

What is the field-strength tensor? Copying QED doesn't work:

$$\partial_\nu B'_\mu - \partial_\mu B'_\nu \neq G(\partial_\nu B_\mu - \partial_\mu B_\nu)G^{-1}.$$

Could write QED case as

$$F_{\mu\nu} = \frac{1}{iq}\left[\mathcal{D}_\nu, \mathcal{D}_\mu\right].$$

Kinetic term for gauge bosons

Candidate for $SU(2)$:

$$F_{\mu\nu} = \frac{1}{ig}\left[\mathcal{D}_\nu, \mathcal{D}_\mu\right] = \partial_\nu B_\mu - \partial_\mu B_\nu + iq\left[B_\nu, B_\mu\right].$$

transforms as required!

$$\mathcal{L}_{\mathrm{YM}} = \bar{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi - \tfrac{1}{2}\mathrm{tr} F_{\mu\nu}F^{\mu\nu}$$

is therefore invariant under local gauge transformations. Mass term $M^2 B_\mu B^\mu$ is incompatible with local gauge invariance, as in electromagnetism, a common nonzero mass for the nucleons is clearly permitted.

In component form, $F^l_{\mu\nu} = \partial_\nu b^l_\mu - \partial_\mu b^l_\nu + g\varepsilon_{jkl} b^j_\mu b^k_\nu$.

General gauge group, $\varepsilon_{jkl} \rightsquigarrow f_{jkl}$.

Formulate electroweak theory

Three crucial clues from experiment:

- Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

- Universal strength of the (charged-current) weak interactions;
- Idealization that neutrinos are massless.

$\boxed{\text{First two clues suggest } SU(2)_L \text{ gauge symmetry}}$

A theory of leptons

$$\mathsf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \mathsf{R} \equiv e_R$$

weak hypercharges $Y_L = -1$, $Y_R = -2$

Gell-Mann–Nishijima connection, $Q = I_3 + \frac{1}{2}Y$

$\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ gauge group $\Rightarrow$ gauge fields:

- weak isovector $\vec{b}_\mu$, coupling g $\boxed{b_\mu^\ell = b_\mu^\ell - \varepsilon_{jk\ell}\alpha^j b_\mu^k - (1/g)\partial_\mu \alpha^\ell}$
- weak isoscalar $\mathcal{A}_\mu$, coupling $g'/2$ $\boxed{\mathcal{A}_\mu \rightarrow \mathcal{A}_\mu - \partial_\mu \alpha}$

Field-strength tensors

$$F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g\varepsilon_{jk\ell} b_\mu^j b_\nu^k \;, SU(2)_L$$

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu \;, U(1)_Y$$

Interaction Lagrangian

$$\boxed{\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}}$$

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4} F^{\ell}_{\mu\nu} F^{\ell\mu\nu} - \tfrac{1}{4} f_{\mu\nu} f^{\mu\nu},$$

$$\begin{aligned}
\mathcal{L}_{\text{leptons}} &= \overline{\mathsf{R}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y\right)\mathsf{R} \\
&+ \overline{\mathsf{L}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_{\mu}\right)\mathsf{L}.
\end{aligned}$$

Mass term $\mathcal{L}_e = -m_e(\bar{e}_{\mathsf{R}} e_{\mathsf{L}} + \bar{e}_{\mathsf{L}} e_{\mathsf{R}}) = -m_e \bar{e} e$ violates local gauge inv.

Theory: 4 massless gauge bosons ($\mathcal{A}_{\mu}\quad b^1_{\mu}\quad b^2_{\mu}\quad b^3_{\mu}$); Nature: 1 ($\gamma$)

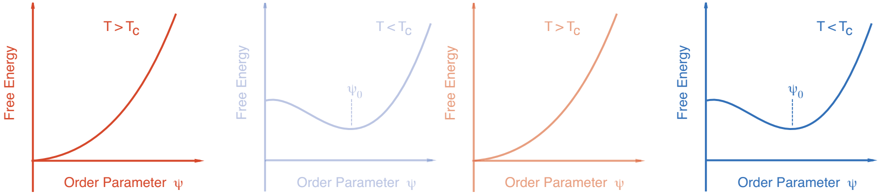
Massive Photon? *Hiding Symmetry*

Recall $\boxed{2}$ miracles of superconductivity:

- No resistance . . . . . . Meissner effect (exclusion of **B**)

Ginzburg–Landau Phenomenology (not a theory from first principles)

normal, resistive charge carriers . . . . . . + superconducting charge carriers



B $= 0$: $$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4$$

$$T > T_c: \quad \alpha > 0 \quad \langle|\psi|^2\rangle_0 = 0$$

$$T < T_c: \quad \alpha < 0 \quad \langle|\psi|^2\rangle_0 \neq 0$$

In a nonzero magnetic field ...

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*}\left| -i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi \right|^2$$

$$\left.\begin{array}{c} e^* = -2 \\ m^* \end{array}\right\} \text{ of superconducting carriers}$$

Weak, slowly varying field: $\psi \approx \psi_0 \neq 0$, $\nabla\psi \approx 0$

Variational analysis $\rightsquigarrow$ wave equation of a *massive photon*

Photon – *gauge boson* – acquires mass

$$\boxed{\lambda^{-1} = e^{\star}|\langle\psi\rangle_0|/\sqrt{m^{\star}c^2}}$$

within superconductor

origin of Meissner effect

Magnet floats (on field lines) above superconductor

Meissner effect levitates Leon Lederman (Snowmass 2001)

Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

- Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

- Add to $\mathcal{L}$ (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

where $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu$ and

$$\boxed{V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda|\,(\phi^\dagger \phi)^2}$$

- Add a Yukawa interaction $\mathcal{L}_{\text{Yukawa}} = -\zeta_e \left[\overline{\mathsf{R}}(\phi^\dagger \mathsf{L}) + (\overline{\mathsf{L}}\phi)\mathsf{R}\right]$

- Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$
  Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{\text{em}}$ invariance

Invariance under $\mathcal{G}$ means $e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0$, so $\mathcal{G}\langle\phi\rangle_0 = 0$

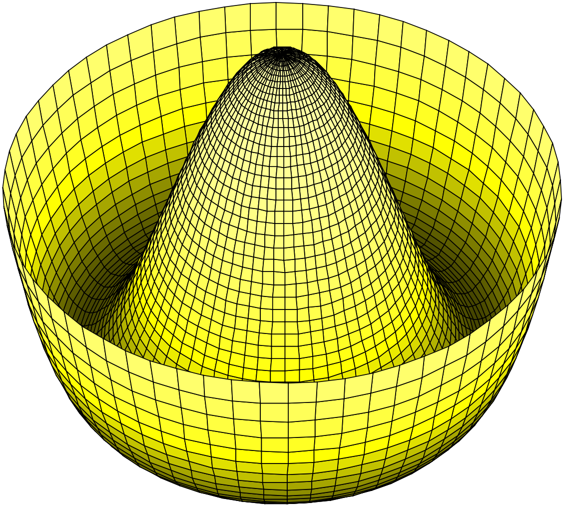
$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$Y\langle\phi\rangle_0 = Y_\phi\langle\phi\rangle_0 = +1\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

Symmetry of laws $\not\Rightarrow$ symmetry of outcomes



Examine electric charge operator Q on the (neutral) vacuum

$$\begin{aligned} Q\langle\phi\rangle_0 &= \tfrac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\ &= \tfrac{1}{2}\begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix}\langle\phi\rangle_0 \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \textit{unbroken!} \end{aligned}$$

Four original generators are broken, *electric charge is not*

- $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y \to U(1)_{\mathrm{em}}$ (will verify)
- Expect massless photon
- Expect gauge bosons corresponding to

$\tau_1,\ \tau_2,\ \frac{1}{2}(\tau_3 - Y) \equiv K$ to acquire masses

Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$; in *unitary gauge*

$$\begin{aligned}\mathcal{L}_{\text{scalar}} &= \tfrac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 \\ &+ \frac{v^2}{8}[g^2 \left| b_\mu^1 - i b_\mu^2 \right|^2 + (g' \mathcal{A}_\mu - g b_\mu^3)^2] \\ &+ \text{interaction terms}\end{aligned}$$

"Higgs boson" η has acquired (mass)^2 $M_H^2 = -2\mu^2 > 0$

$$\text{Define } W_\mu^\pm = \frac{b_\mu^1 \mp i b_\mu^2}{\sqrt{2}}$$

$$\frac{g^2 v^2}{8}(\left| W_\mu^+ \right|^2 + \left| W_\mu^- \right|^2) \Longleftrightarrow M_{W^\pm} = gv/2$$

$(v^2/8)(g'\mathcal{A}_\mu - g b_\mu^3)^2 \ldots$

Now define orthogonal combinations

$$Z_\mu = \frac{-g'\mathcal{A}_\mu + g b_\mu^3}{\sqrt{g^2 + g'^2}} \qquad A_\mu = \frac{g\mathcal{A}_\mu + g' b_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$M_{Z^0} = \sqrt{g^2 + g'^2}\, v/2 = M_W \sqrt{1 + g'^2/g^2}$$

$$\boxed{A_\mu \text{ remains massless}}$$

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\zeta_e \frac{(v+\eta)}{\sqrt{2}}(\bar{e}_\mathrm{R} e_\mathrm{L} + \bar{e}_\mathrm{L} e_\mathrm{R}) \\
&= -\frac{\zeta_e v}{\sqrt{2}}\bar{e}e - \frac{\zeta_e \eta}{\sqrt{2}}\bar{e}e
\end{aligned}
$$

electron acquires $m_e = \zeta_e v/\sqrt{2}$

$\boxed{\text{Higgs-boson coupling to electrons: } m_e/v \quad (\propto \text{mass})}$

Desired particle content . . . plus a Higgs scalar

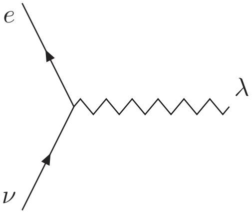
Values of couplings, electroweak scale v?

What about interactions?

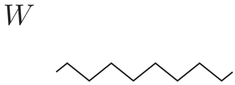
Interactions ...

$$\mathcal{L}_{W\text{-}\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^{\mu}(1-\gamma_5)eW_{\mu}^{+} + \bar{e}\gamma^{\mu}(1-\gamma_5)\nu W_{\mu}^{-}]$$

$+$ similar terms for μ and τ



$$\frac{-ig}{2\sqrt{2}}\gamma_{\lambda}(1-\gamma_5)$$



$$= \frac{-i(g_{\mu\nu} - k_{\mu}k_{\nu}/M_W^2)}{k^2 - M_W^2}\ .$$

Compute $\nu_\mu e \to \mu\nu_e$

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu / M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\frac{g^4}{16 M_W^4} = 2G_F^2 \Rightarrow \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Using $M_W = gv/2$, determine the electroweak scale

$$\boxed{v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}}$$

$$\Rightarrow \langle\phi^0\rangle_0 = (G_F\sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

W-propagator modifies HE behavior

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \to \infty} \sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

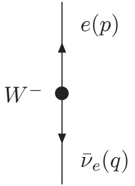
Partial-wave unitarity respected for

$$s < M_W^2[\exp(\pi\sqrt{2}/G_F M_W^2) - 1]$$

W-boson properties

No prediction yet for M_W (haven't determined g)

Leptonic decay $W^- \to e^-\nu_e$



$$p \approx \left(\frac{M_W}{2}; \frac{M_W \sin\theta}{2}, 0, \frac{M_W \cos\theta}{2}\right)$$

$$q \approx \left(\frac{M_W}{2}; -\frac{M_W \sin\theta}{2}, 0, -\frac{M_W \cos\theta}{2}\right)$$

$$\mathcal{M} = -i \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{u}(e,p)\gamma_\mu(1-\gamma_5)v(\nu,q)\,\varepsilon^\mu$$

$\varepsilon^\mu = (0;\hat{\varepsilon})$: W polarization vector in its rest frame

$$\begin{aligned} |\mathcal{M}|^2 &= \frac{G_F M_W^2}{\sqrt{2}} \mathrm{tr}\left[\not{\varepsilon}(1-\gamma_5)\not{q}(1+\gamma_5)\not{\varepsilon}^*\not{p}\right] \; ; \\ \mathrm{tr}[\cdots] &= \left[\varepsilon\cdot q\,\varepsilon^*\cdot p - \varepsilon\cdot\varepsilon^*\,q\cdot p + \varepsilon\cdot p\,\varepsilon^*\cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma\right] \end{aligned}$$

$$\mathrm{tr}[\cdots] = [\varepsilon \cdot q\, \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^*\, q \cdot p + \varepsilon \cdot p\, \varepsilon^* \cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma]$$

decay rate is independent of W polarization; look first at longitudinal pol.
$\varepsilon^\mu = (0; 0, 0, 1) = \varepsilon^{*\mu}$, eliminate $\epsilon_{\mu\nu\rho\sigma}$

$$|\mathcal{M}|^2 = \frac{4G_F M_W^4}{\sqrt{2}} \sin^2\theta$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2} \frac{\mathcal{S}_{12}}{M_W^3}$$

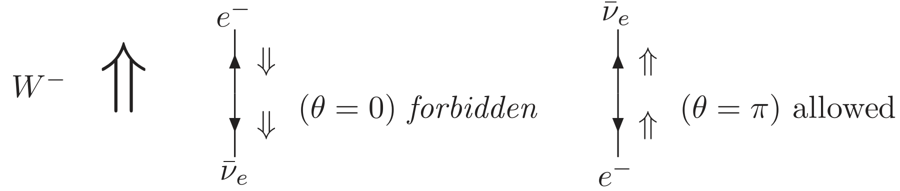
$$\mathcal{S}_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \sin^2\theta \qquad \boxed{\Gamma(W \to e\nu) = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

Other helicities: $\varepsilon^{\mu}_{\pm 1} = (0; -1, \mp i, 0)/\sqrt{2}$

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1 \mp \cos\theta)^2$$

Extinctions at $\cos\theta = \pm 1$ are consequences of angular momentum conservation:



(situation reversed for $W^+ \to e^+\nu_e$)

e^+ follows polarization direction of W^+

e^- avoids polarization direction of W^-

important for discovery of $W^\pm$ in $\bar{p}p$ $(\bar{q}q)$ C violation

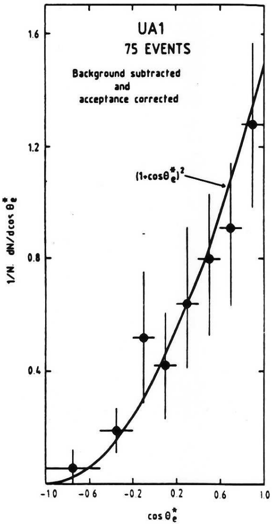


Fig. 2. The W decay angular distribution of the emission angle θ^* of the electron (positron) with respect to the proton (antiproton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the $(V-A)$ expectation of $(1+\cos\theta^*)^2$.

Interactions . . .

$$\mathcal{L}_{A\text{-}\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu e A_\mu$$

. . . vector interaction; $\Rightarrow$ A_μ as γ, provided we identify

$$\boxed{gg'/\sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan\theta_W$

θ_W: weak mixing angle

$$\begin{aligned} g &= e/\sin\theta_W \geq e \\ g' &= e/\cos\theta_W \geq e \end{aligned}$$

$$Z_\mu = b_\mu^3 \cos\theta_W - \mathcal{A}_\mu \sin\theta_W \quad A_\mu = \mathcal{A}_\mu \cos\theta_W + b_\mu^3 \sin\theta_W$$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu}\gamma^\mu(1-\gamma_5)\nu \, Z_\mu$$

Purely left-handed!

Interactions . . .

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W}\bar{e}\left[L_e\gamma^\mu(1-\gamma_5) + R_e\gamma^\mu(1+\gamma_5)\right] e\, Z_\mu$$

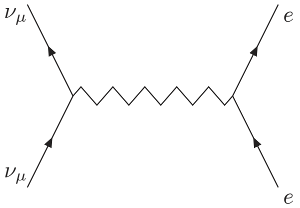
$$\begin{aligned} L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \\ R_e &= 2\sin^2\theta_W = 2x_W \end{aligned}$$

Z-decay calculation analogous to $W^\pm$

$$\begin{aligned} \Gamma(Z \to \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \to e^+e^-) &= \Gamma(Z \to \nu\bar{\nu})\left[L_e^2 + R_e^2\right] \end{aligned}$$

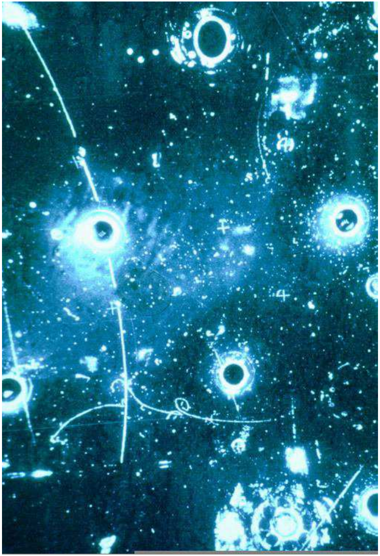
Neutral-current interactions

New νe reaction, not present in $V - A$



$$\begin{aligned}
\sigma(\nu_\mu e \to \nu_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2 + R_e^2/3\right] \\
\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2/3 + R_e^2\right] \\
\sigma(\nu_e e \to \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2 + R_e^2/3\right] \\
\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2/3 + R_e^2\right]
\end{aligned}$$

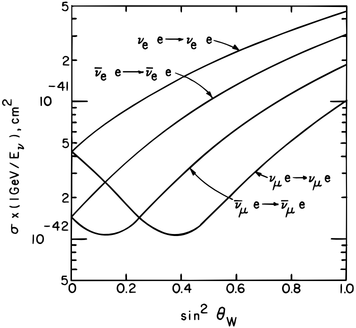
Gargamelle $\nu_\mu e$ event (1973)



- Electromagnetism is mediated by a massless photon, coupled to the electric charge;
- Mediator of charged-current weak interaction acquires a mass $M_W^2 = \pi\alpha / G_F\sqrt{2}\sin^2\theta_W$,
- Mediator of (new!) neutral-current weak interaction acquires mass $M_Z^2 = M_W^2/\cos^2\theta_W$;
- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;
- Fermions can acquire mass—values not predicted.

Determine $\sin^2\theta_W$ to predict M_W, M_Z

“Model-independent” analysis

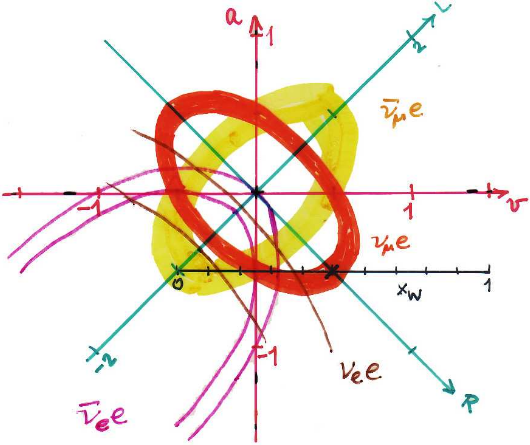


Measure all cross sections to determine chiral couplings L_e and R_e or traditional vector and axial couplings v and a

$$a = \tfrac{1}{2}(L_e - R_e) \qquad v = \tfrac{1}{2}(L_e - R_e)$$
$$L_e = v + a \qquad R_e = v - a$$

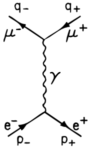
model-independent in V, A framework

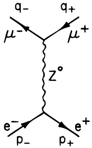
Neutrino–electron scattering



Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ ($v \leftrightarrow a$)

Consider $e^+e^- \rightarrow \mu^+\mu^-$





$$
\begin{aligned}
\mathcal{M} \;=\; & -ie^2\bar{u}(\mu, q_-)\gamma_\lambda Q_\mu v(\mu, q_+)\frac{g^{\lambda\nu}}{s}\bar{v}(e, p_+)\gamma_\nu u(e.p_-) \\
& + \frac{i}{2}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)\bar{u}(\mu, q_-)\gamma_\lambda[R_\mu(1+\gamma_5) + L_\mu(1-\gamma_5)]v(\mu, q_+) \\
& \times \frac{g^{\lambda\nu}}{s - M_Z^2}\bar{v}(e, p_+)\gamma_\nu[R_e(1+\gamma_5) + L_e(1-\gamma_5)]u(e, p_-)
\end{aligned}
$$

muon charge $Q_\mu = -1$

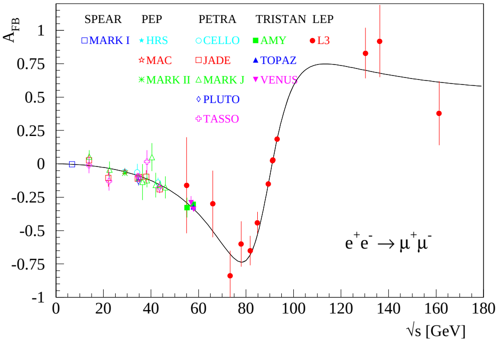
$e^+e^- \rightarrow \mu^+\mu^-$ ...

$$
\begin{aligned}
\frac{d\sigma}{dz} = & \frac{\pi\alpha^2 Q_\mu^2}{2s}(1+z^2) \\
& -\frac{\alpha Q_\mu G_F M_Z^2(s-M_Z^2)}{8\sqrt{2}[(s-M_Z^2)^2+M_Z^2\Gamma^2]} \\
& \times[(R_e+L_e)(R_\mu+L_\mu)(1+z^2)+2(R_e-L_e)(R_\mu-L_\mu)z] \\
& +\frac{G_F^2 M_Z^4 s}{64\pi[(s-M_Z^2)^2+M_Z^2\Gamma^2]} \\
& \times[(R_e^2+L_e^2)(R_\mu^2+L_\mu^2)(1+z^2)+2(R_e^2-L_e^2)(R_\mu^2-L_\mu^2)z]
\end{aligned}
$$

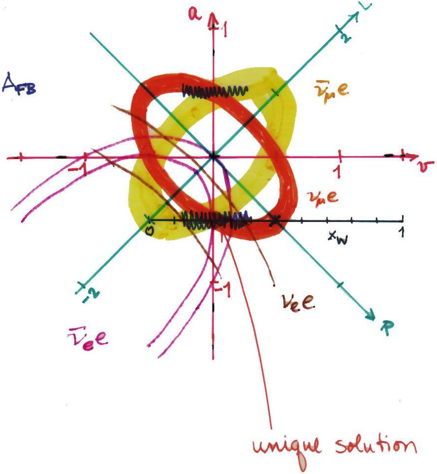
Measuring A resolves ambiguity

$$\text{Forward-backward asymmetry } A \equiv \frac{\int_0^1 dz\, d\sigma/dz - \int_{-1}^0 dz\, d\sigma/dz}{\int_{-1}^1 dz\, d\sigma/dz}$$

$$\begin{aligned} \lim_{s/M_Z^2 \ll 1} A &= \frac{3G_F s}{16\pi\alpha Q_\mu \sqrt{2}}(R_e - L_e)(R_\mu - L_\mu) \\ &\approx -6.7 \times 10^{-5} \left(\frac{s}{1\ \mathrm{GeV}^2}\right)(R_e - L_e)(R_\mu - L_\mu) = -3G_F s a^2/4\pi\alpha\sqrt{2} \end{aligned}$$



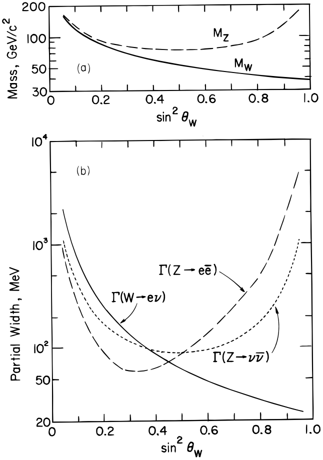
Neutrino–electron scattering $e^+e^- \to \mu^+\mu^-$



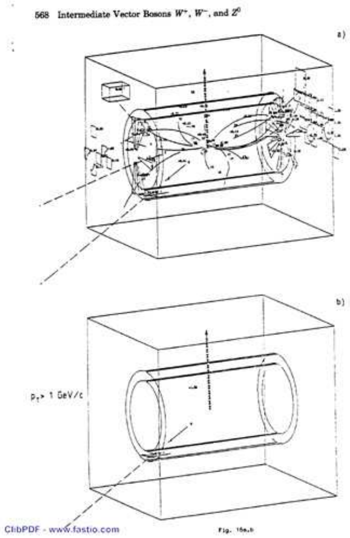
$\boxed{\text{Validate EW theory, measure } \sin^2\theta_W}$

With a measurement of $\sin^2\theta_W$, predict

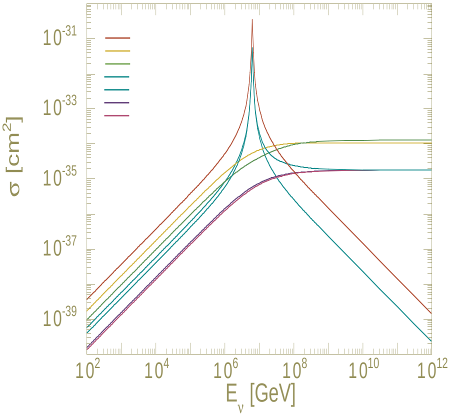
$M_W^2 = \pi\alpha / G_F\sqrt{2}\sin^2\theta_W \approx (37.28\ \text{GeV}/c^2)^2/\sin^2\theta_W$ $\quad M_Z^2 = M_W^2/\cos^2\theta_W$



First Z from UA1



νe cross sections . . .



At low energies: $\sigma(\bar{\nu}_e e \to$ hadrons$) > \sigma(\nu_\mu e \to \mu\nu_e) > \sigma(\nu_e e \to \nu_e e) > \sigma((\bar{\nu}_e e \to \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \to \bar{\nu}_e e) > \sigma(\nu_\mu e \to \nu_\mu e) > \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$

Electroweak interactions of quarks

- Left-handed doublet

$$\begin{array}{lccc} & I_3 & Q & Y = 2(Q - I_3) \\ \mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L & \begin{array}{c} \frac{1}{2} \\ -\frac{1}{2} \end{array} & \begin{array}{c} +\frac{2}{3} \\ -\frac{1}{3} \end{array} & \frac{1}{3} \end{array}$$

- two right-handed singlets

$$\begin{array}{lccc} & I_3 & Q & Y = 2(Q - I_3) \\ \mathsf{R}_u = u_R & 0 & +\frac{2}{3} & +\frac{4}{3} \\ \mathsf{R}_d = d_R & 0 & -\frac{1}{3} & -\frac{2}{3} \end{array}$$

Electroweak interactions of quarks

- CC interaction

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{u}\gamma^\mu(1-\gamma_5)d \; W_\mu^+ + \bar{d}\gamma^\mu(1-\gamma_5)u \; W_\mu^- \right]$$

identical in form to $\mathcal{L}_{W\text{-}\ell}$: universality $\Leftrightarrow$ weak isospin

- NC interaction

$$\mathcal{L}_{Z\text{-}q} = \frac{-g}{4\cos\theta_W} \sum_{i=u,d} \bar{q}_i\gamma^\mu \left[L_i(1-\gamma_5) + R_i(1+\gamma_5) \right] q_i \; Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \qquad R_i = -2Q_i \sin^2\theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z\text{-}\ell}$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ *not quite right*

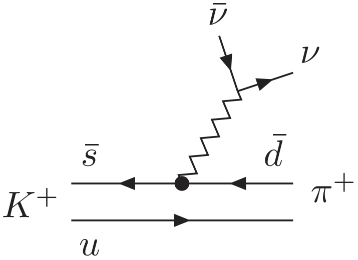
$$\text{Good:} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \text{Better:} \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d\,\cos\theta_C + s\,\sin\theta_C \qquad \cos\theta_C = 0.9736 \pm 0.0010$$

"Cabibbo-rotated" doublet perfects CC interaction (up to small third-generation effects) but $\Rightarrow$ serious trouble for NC

$$\begin{aligned} \mathcal{L}_{Z\text{-}q} \;=\; & \frac{-g}{4\cos\theta_W} Z_\mu \left\{ \bar{u}\gamma^\mu \left[L_u(1-\gamma_5) + R_u(1+\gamma_5) \right] u \right. \\ & + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d\,\cos^2\theta_C \\ & + \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s\,\sin^2\theta_C \\ & + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s\,\sin\theta_C\cos\theta_C \\ & \left. + \,\bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d\,\sin\theta_C\cos\theta_C \right\} \end{aligned}$$

Strangeness-changing NC interactions highly suppressed!



BNL E-787/E-949 has three $K^+ \to \pi^+ \nu \bar{\nu}$ candidates, with $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) = 1.47^{+1.30}_{-0.89} \times 10^{-10}$

Phys. Rev. Lett. **93**, 031801 (2004)

(SM: 0.78 ± 0.11: U. Haisch, hep-ph/0605170)

Glashow–Iliopoulos–Maiani

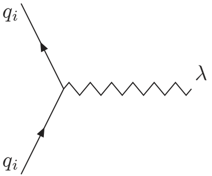
two LH doublets: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$

$$(s_\theta = s \cos\theta_C - d \sin\theta_C)$$

$+$ right-handed singlets, e_R, μ_R, u_R, d_R, c_R, s_R

$\boxed{\text{Required new charmed quark, } c}$

Cross terms vanish in $\mathcal{L}_{Z\text{-}q}$,



$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \quad R_i = -2Q_i \sin^2\theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\mathcal{O}\Psi \; W_{\mu}^{+} + \text{h.c.}\right]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}$

flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U: unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z\text{-}q}^{\text{iso}} = \dfrac{-g}{4\cos\theta_W}\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\left[\mathcal{O},\mathcal{O}^{\dagger}\right]\Psi$

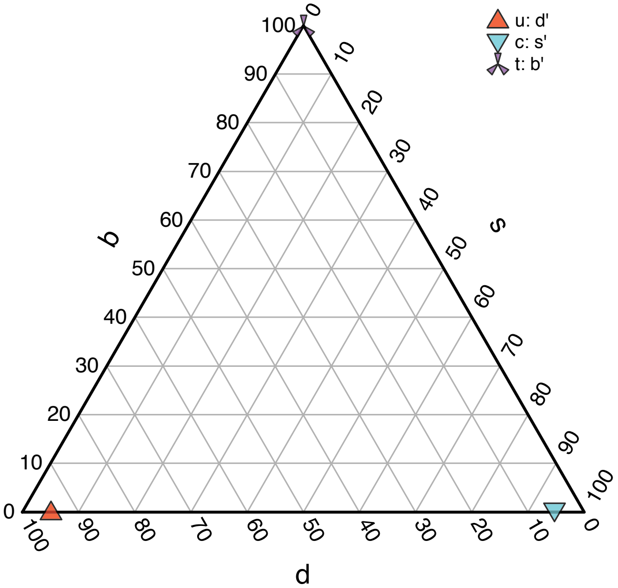
Since $\left[\mathcal{O},\mathcal{O}^{\dagger}\right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\Rightarrow$ NC interaction is flavor-diagonal

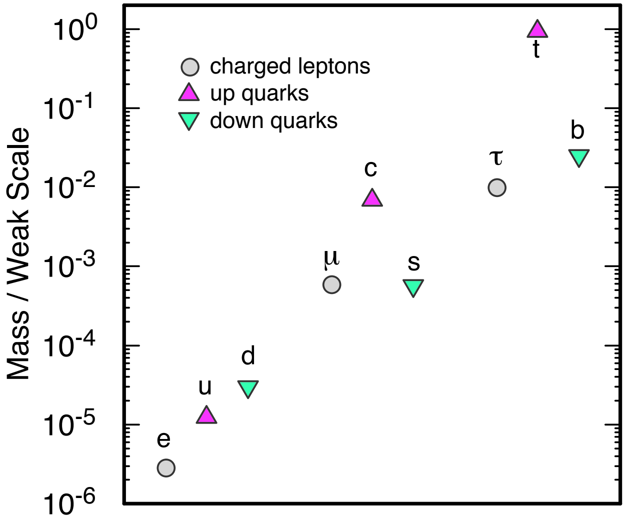
General $n \times n$ mixing matrix U: $n(n-1)/2$ real $\angle$, $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi-Maskawa): $3\,\angle + 1$ phase $\Rightarrow$ CP violation

Quark Mixing (Components: $|U_{u\alpha}|^2$, etc.)



Quark & charged-lepton masses



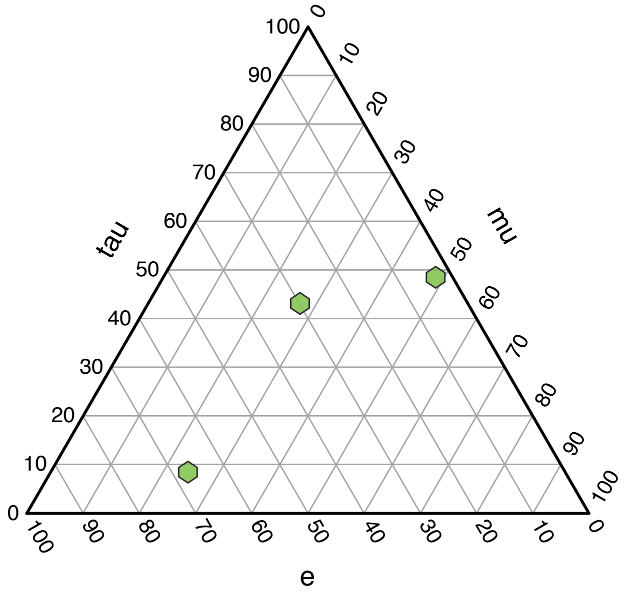
If neutrinos have Dirac masses, ν Yukawa couplings $\lesssim 10^{-11}$

Fermion mass is accommodated, not explained

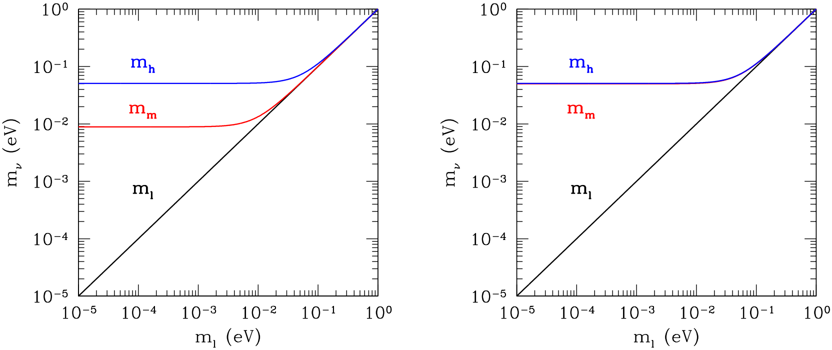
- All fermion masses $\sim$ physics beyond the standard model!
- $\zeta_t \approx 1 \qquad \zeta_e \approx 3 \times 10^{-6} \qquad \zeta_\nu \approx 10^{-11}$??

  What accounts for the range and values of the Yukawa couplings?
- There may be *other sources* of neutrino mass

Neutrino Mixing (representative θ_{12}, θ_{23} values, $\theta_{13} = 10^\circ$)



Absolute scale of neutrino masses is not yet known



Normal spectrum | Inverted spectrum

$m_2^2 - m_1^2 = \Delta m_\odot^2 = 7.9 \times 10^{-5}\ \mathrm{eV}^2$ $\quad |m_3^2 - m_1^2| = \Delta m_{\mathrm{atm}}^2 = 2.5 \times 10^{-3}\ \mathrm{eV}^2$

$$\rho_c \equiv 3H_0^2/8\pi G_N = 1.05h^2 \times 10^4\ \mathrm{eV\ cm}^{-3} = 5.6 \times 10^3\ \mathrm{eV\ cm}^{-3}$$

$$\boxed{(56\nu_i + 56\bar{\nu}_i)\ \mathrm{cm}^{-3} \rightsquigarrow \sum_i m_{\nu_i} \lesssim 50\ \mathrm{eV}}$$

Successful predictions of $SU(2)_L \otimes U(1)_Y$ theory:

- neutral-current interactions
- necessity of charm
- existence and properties of $W^\pm$ and Z^0

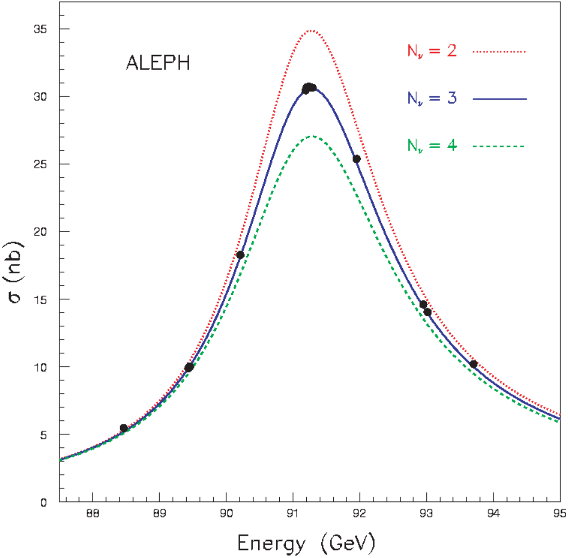
$+$ a decade of precision EW tests (one-per-mille)

M_W	$80\,398 \pm 25$ MeV
Γ_W	$2\,140 \pm 60$ MeV
M_Z	$91\,187.6 \pm 2.1$ MeV
Γ_Z	2495.2 ± 2.3 MeV
$\sigma^0_{\text{hadronic}}$	41.541 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

$$\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$$

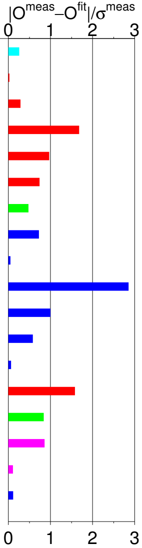
light ν : $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i\bar{\nu}_i) = 2.994 \pm 0.012 \quad (\nu_e, \nu_\mu, \nu_\tau)$

Three light neutrinos



Measurement–best-fit differences . . .

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.743
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21581
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.377
Γ_W [GeV]	2.097 ± 0.048	2.092
m_t [GeV]	172.6 ± 1.4	172.8



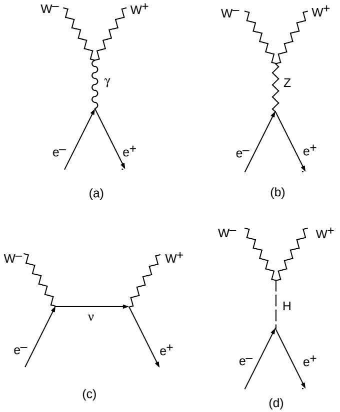
March 2008

LEP Electroweak Working Group, Winter 2008

Why a Higgs boson must exist

▷ Role in canceling high-energy divergences

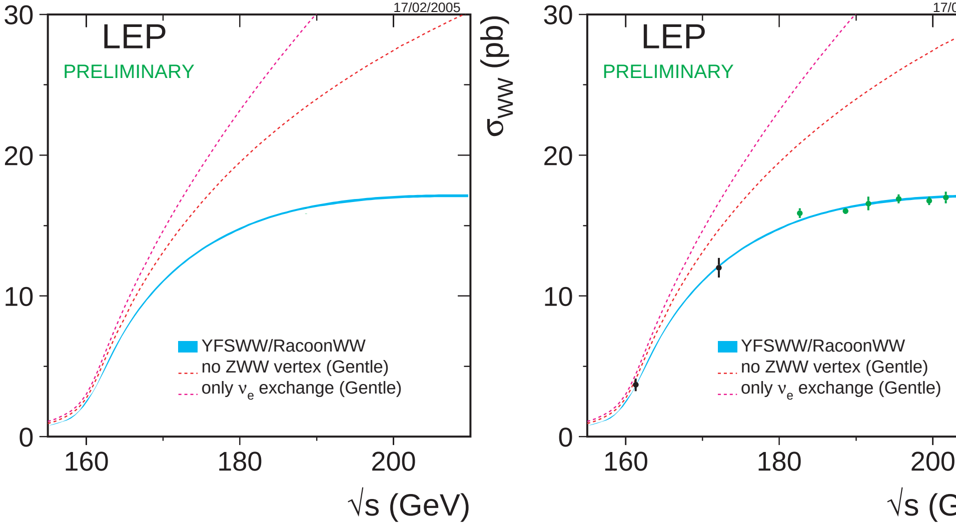
S-matrix analysis of $e^+e^- \to W^+W^-$



Individual $J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$ have unacceptable high-energy behavior ($\propto s$)

. . . But sum is well-behaved

"Gauge cancellation" observed at LEP2 (Tevatron)



$J = 0$ amplitude exists because electrons have mass, and can be found in "wrong" helicity state

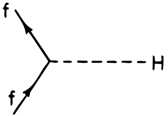
$$\mathcal{M}_{\nu}^{(0)} \propto s^{\frac{1}{2}} \; \text{: unacceptable HE behavior}$$

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$$\boxed{\Rightarrow He\bar{e} \text{ coupling must be} \propto m_e,}$$

because "wrong-helicity" amplitudes $\propto m_e$



$$\frac{-im_f}{v} = -im_f(G_F\sqrt{2})^{1/2}$$

If the Higgs boson did not exist, something else would have to cure divergent behavior

If gauge symmetry were unbroken . . .

- no Higgs boson
- no longitudinal gauge bosons
- no extreme divergences
- no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

- gauge structure of couplings eliminates the most severe divergences
- lesser—but potentially fatal—divergence arises because the electron has mass . . . due to the Higgs mechanism
- SSB provides its own cure—the Higgs boson

Similar interplay & compensation *must exist* in any acceptable theory

The importance of the 1-TeV scale

EW theory does not predict Higgs-boson mass

▷ Conditional *upper bound* from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)$$

Most channels decouple – pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies) – $\forall M_H$.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0$$

L: longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,[1] s-wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \to \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect pw unitarity condition $|a_0| \leq 1$

$$\Longrightarrow \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} = 1 \text{ TeV}/c^2$$

condition for perturbative unitarity

[1] Convenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\text{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2/\sqrt{2}$.

- If the bound is respected
  - weak interactions remain weak at all energies
  - perturbation theory is everywhere reliable

- If the bound is violated
  - perturbation theory breaks down
  - weak interactions among $W^{\pm}$, Z, H become strong on 1-TeV scale

$\Rightarrow$ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

Threshold behavior of the pw amplitudes a_{IJ} follows from chiral symmetry

$$
\begin{array}{lll}
a_{00} \approx & G_F s/8\pi\sqrt{2} & \text{attractive} \\
a_{11} \approx & G_F s/48\pi\sqrt{2} & \text{attractive} \\
a_{20} \approx & -G_F s/16\pi\sqrt{2} & \text{repulsive}
\end{array}
$$

Lee, Quigg, Thacker, *Phys. Rev. D* **16,** 1519 (1977)

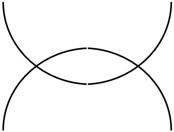
Bounding M_H from above . . .

Triviality of scalar field theory

- Only *noninteracting* scalar field theories make sense on all energy scales
- Quantum field theory vacuum is a dielectric medium that screens charge
- $\Rightarrow$ *effective charge* is a function of the distance or, equivalently, of the energy scale

$$\boxed{\text{running coupling constant}}$$

In $\lambda\phi^4$ theory, calculate variation of coupling constant λ in perturbation theory by summing bubble graphs



$\lambda(\mu)$ is related to a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log\left(\Lambda/\mu\right)$$

(Perturbation theory reliable only when λ is small, lattice field theory treats strong-coupling regime)

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to $-\infty$), *require* $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log\left(\Lambda/\mu\right)$$

...implies an *upper bound*

$$\boxed{\lambda(\mu) \leq 2\pi^2/3 \log\left(\Lambda/\mu\right)}$$

If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \to \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the $\boxed{\text{bound}}$ forces $\lambda(\mu)$ to zero.

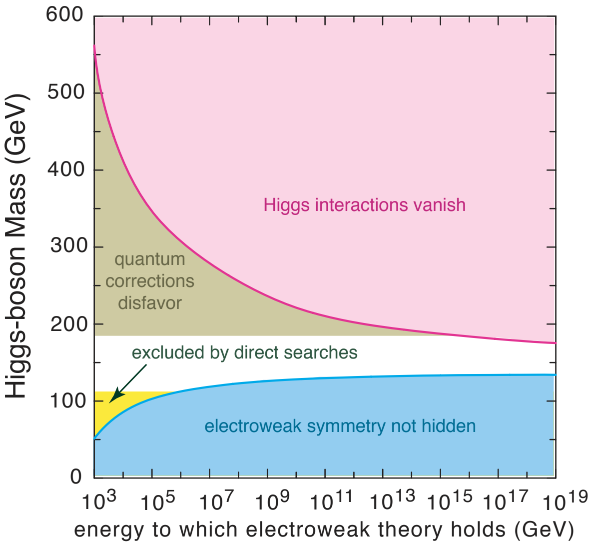
$\longrightarrow$ free field theory "trivial"

Rewrite as bound on M_H:

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\boxed{\Lambda \leq M_H \exp\left(4\pi^2 v^2/3M_H^2\right)}$$


Higgs-boson Mass (GeV)
600
500
400
300
200
100
0
Higgs interactions vanish
quantum corrections disfavor
excluded by direct searches
electroweak symmetry not hidden
10^3 10^5 10^7 10^9 10^{11} 10^{13} 10^{15} 10^{17} 10^{19}
energy to which electroweak theory holds (GeV)

Moral: For any M_H, there is a *maximum energy scale* $\Lambda^\star$ at which the theory ceases to make sense.

The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when $M_H \to 1\ \mathrm{TeV}/c^2$ and interactions become strong

Lattice analyses $\Longrightarrow M_H \lesssim 710 \pm 60\ \mathrm{GeV}/c^2$ if theory describes physics to a few percent up to a few TeV

$\boxed{\text{If } M_H \to 1 \text{ TeV EW theory lives on brink of instability}}$

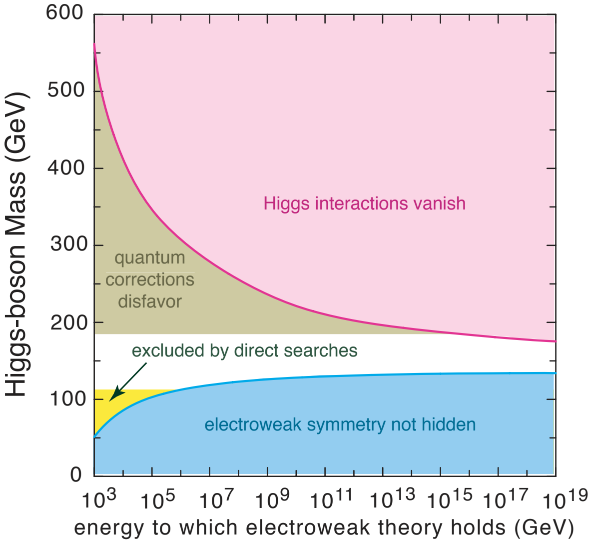
Lower bound by requiring EWSB vacuum $V(v) < V(0)$

Requiring that $\langle\phi\rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition . . . (for $m_t \lesssim M_W$)

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2}(2M_W^4 + M_Z^4 - 4m_t^4)\log(\Lambda^2/v^2)$$

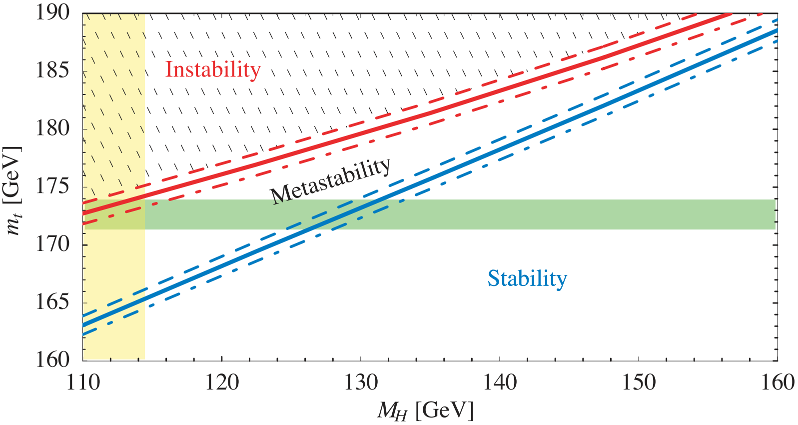
(No illuminating analytic form for heavy m_t)

If the Higgs boson is relatively light (which would require explanation) then the theory can be self-consistent up to very high energies



If EW theory is to make sense all the way up to a unification scale $\Lambda^\star = 10^{16}$ GeV, then 134 GeV/$c^2 \lesssim M_H \lesssim$ 177 GeV

Metastability of the standard-model vacuum?



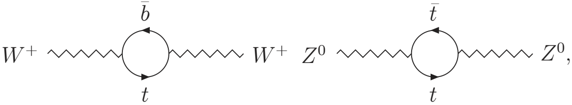
$m_t = 172.6 \pm 1.4$ GeV

Isidori, et al., hep-ph/0104016

Experimental clues to the Higgs-boson mass

Sensitivity of EW observables to m_t gave early indications for massive top

Quantum corrections to SM predictions for M_W and M_Z arise from different quark loops



$$\ldots\text{alter the link}\quad \underbrace{M_W^2}_{(80.398 \pm 0.025\ \text{GeV})^2} = \underbrace{M_Z^2\left(1-\sin^2\theta_W\right)}_{(80.939\ \text{GeV})^2}(1-\Delta\rho)$$

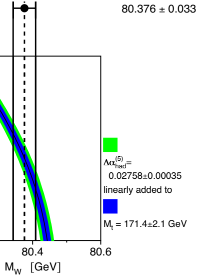
where $\Delta\rho \approx \Delta\rho^{(\text{quarks})} \approx 3G_F m_t^2/8\pi^2\sqrt{2}$

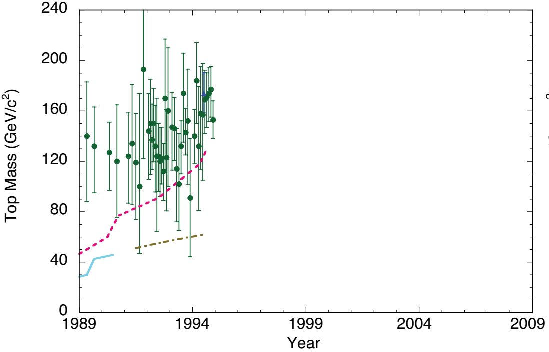
Strong dependence on m_t^2 accounts for precision of m_t estimates derived from EW observables

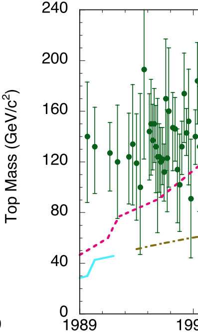
Tevatron: $\delta m_t/m_t \approx 0.8\%$... Look beyond quark loops to next most important quantum corrections: $\boxed{\textit{Higgs-boson effects}}$

Global fits to precision EW measurements

- precision improves with time / calculations improve with time







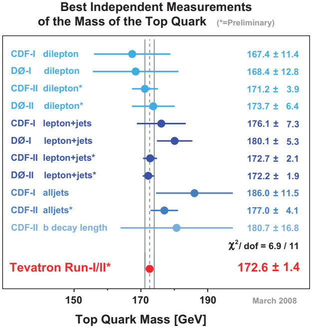
11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19}$ GeV/c^2

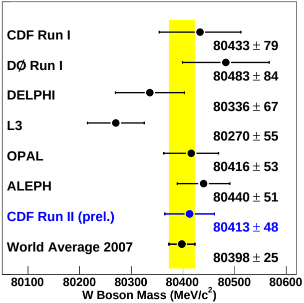
Direct measurements (2008): $m_t = 172.6 \pm 1.4$ GeV/c^2

H quantum corrections smaller than t corrections, exhibit more subtle dependence on M_H than the m_t^2 dependence of the top-quark corrections

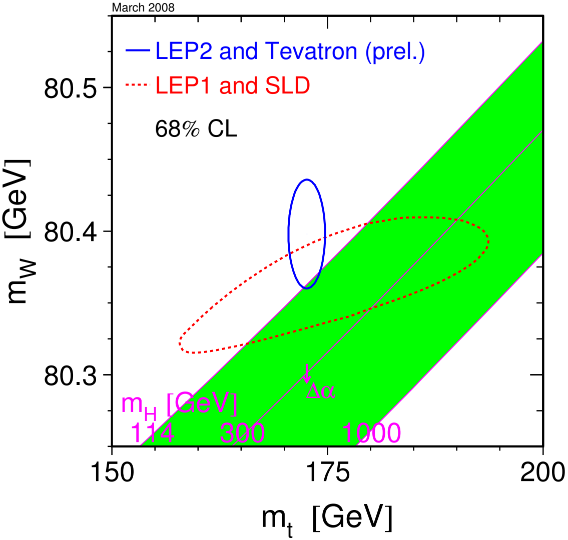
$$\Delta\rho^{(\mathrm{Higgs})} = \mathcal{C} \cdot \ln\left(\frac{M_H}{v}\right)$$

M_Z known to 23 ppm, m_t and M_W well measured





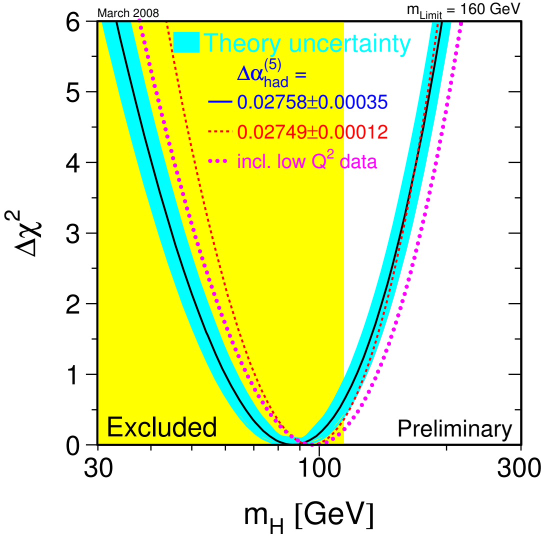
. . . so examine dependence of M_W upon m_t and M_H



Direct, indirect determinations agree reasonably

Both favor a light Higgs boson, . . . *within framework of SM analysis.*

Fit to a universe of data



Standard-Model $M_H \lesssim 190$ GeV at 95% CL

- *Within SM,* LEP EWWG deduce a 95% CL upper limit, $M_H \lesssim 190\ \mathrm{GeV}/c^2$.
- Direct searches at LEP $\Rightarrow M_H > 114.4\ \mathrm{GeV}/c^2$, excluding much of the favored region
- Either the Higgs boson is just around the corner, or SM analysis is misleading

Things will soon be popping!

- Tevatron, LHC measurements will determine m_t within ≈ 1 GeV
  ... and improve δM_W to about 15 MeV
- As the Tevatron's integrated luminosity approaches 10 fb^{-1}, CDF and DØ will explore the region of M_H not excluded by LEP
- ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC;
  could require a few years, at low mass;
  full range accessible, $\gamma\gamma, \ell\ell\nu\nu, b\bar{b}, \ell^+\ell^-\ell^+\ell^-, \ell\nu jj, \tau\tau$ channels.

Information on Higgs properties & search, [Slide 152]

EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of superconducting phase transition;

. . . had to introduce new, elementary scalars

GL is not the last word on superconductivity:

dynamical Bardeen–Cooper–Schrieffer theory

The elementary fermions – electrons – and gauge interactions – QED – needed to generate the scalar bound states are already present in the case of superconductivity.

Could a scheme of similar economy account for EWSB?

Chiral symmetry (global)

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}_f^c \left[i\gamma^\mu \mathcal{D}_\mu \mathbf{I} - \mathbf{m} \right] q_f^c - \tfrac{1}{2}\mathrm{tr}\left(G_{\mu\nu} G^{\mu\nu} \right)$$

$$\mathbf{m} = \begin{pmatrix} m_u & 0 & 0 & 0 \\ 0 & m_d & 0 & 0 \\ 0 & 0 & m_s & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

Mass terms connect left & right:

$$\begin{aligned} m\,\bar{q}^c q^c &= m\bar{q}^c \left[\tfrac{1}{2}(1+\gamma_5) + \tfrac{1}{2}(1-\gamma_5) \right] q^c \\ &= m\left[\bar{q}_L^c q_R^c + \bar{q}_R^c q_L^c \right] \end{aligned}$$

n massless quarks $\rightsquigarrow \mathrm{SU}(n)_L \otimes \mathrm{SU}(n)_R$ chiral symmetry

Chiral symmetry . . .

n massless quarks $\rightsquigarrow \mathrm{SU}(n)_\mathrm{L} \otimes \mathrm{SU}(n)_\mathrm{R}$ chiral symmetry

Soft-pion theorems: $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{SU}(2)_\mathrm{R}$ chiral symmetry

Excellent approximation: $m_u, m_d \to 0$

Current algebra: $\mathrm{SU}(3)_\mathrm{L} \otimes \mathrm{SU}(3)_\mathrm{R}$ chiral symmetry

Good approximation: $m_u, m_d, m_s \to 0$

$\boxed{\text{Infer very light } u, d + \text{not-as-light } s}$

$m_u \approx 3$ MeV, $m_d \approx 6$ MeV, $m_s \approx 100$ MeV

$\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ + massless u and d
(treat $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ as perturbation)

QCD has exact $\mathrm{SU(2)_L} \otimes \mathrm{SU(2)_R}$ chiral symmetry.

At an energy scale $\sim \Lambda_{\mathrm{QCD}}$, strong interactions become strong, fermion condensates $\langle \bar{q}q \rangle$ appear, and

$$\mathrm{SU(2)_L} \otimes \mathrm{SU(2)_R} \to \mathrm{SU(2)_V}$$

$\rightsquigarrow$ 3 Goldstone bosons, one for each broken generator:
3 massless pions (Nambu)

$\mathrm{SU(2)_V}$ (residual) isospin symmetry broken by EM, $m_d - m_u$

The irony of isospin: Motivated Yang-Mills theory, seems an accident of $m_u \approx m_d \approx 0$*, external to QCD!*

Broken generators: 3 axial currents; couplings to π measured by pion decay constant f_π.

Turn on $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$: EW gauge bosons couple to axial currents, acquire masses of order $\sim g f_\pi$.

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4} \quad (W^+, W^-, W_3, \mathcal{A})$$

same structure as standard EW theory.

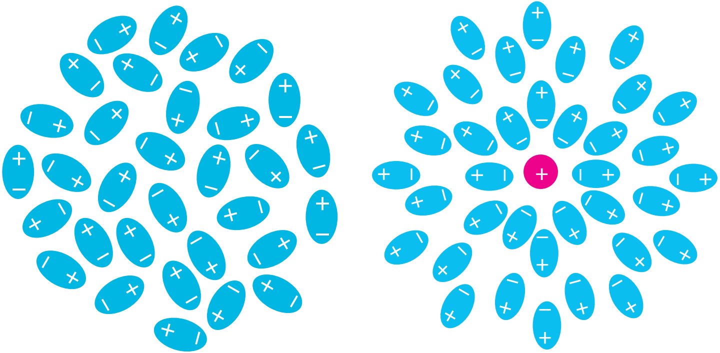
Diagonalize: $M_W^2 = g^2 f_\pi^2/4$, $M_Z^2 = (g^2 + g'^2) f_\pi^2/4$, $M_A^2 = 0$, so

$$\frac{M_Z^2}{M_W^2} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2\theta_W}$$

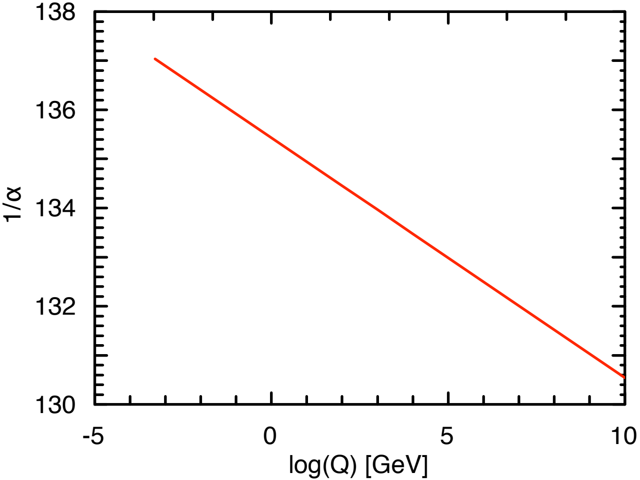
Massless pions disappear from physical spectrum, to become longitudinal components of weak bosons. $\boxed{M_W \approx 30\ \mathrm{MeV}/c^2}$ No fermion masses ...

Charge screening in electrodynamics

Dielectric (polarizable) medium . . .

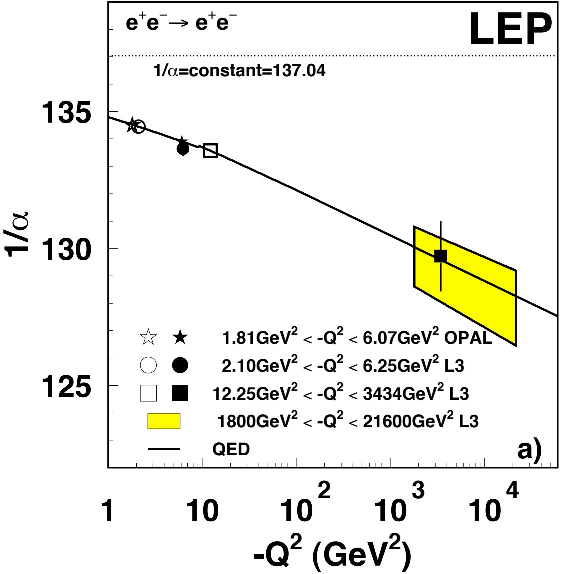


Charge screening in QED (electrons + photons)



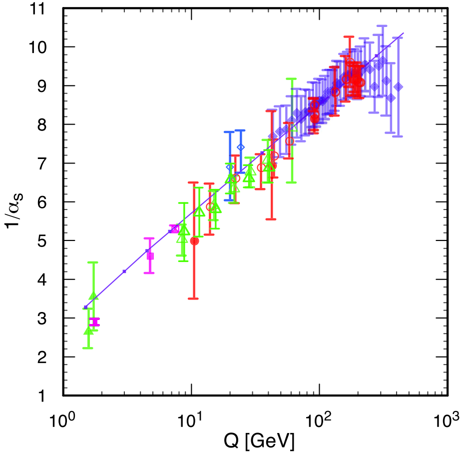
$$1/\alpha(Q) = 1/\alpha_0 - \frac{2}{3\pi} \ln\left(\frac{Q}{m}\right)$$

Charge screening in QED (real world)



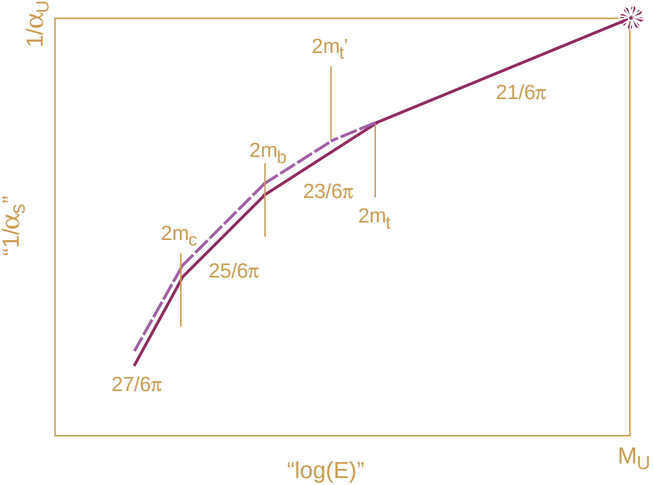
Color *antiscreening* in QCD

Screening from $q\bar{q}$ pairs, camouflage from gluon cloud



$$\frac{1}{\alpha_s(Q)} = \frac{1}{\alpha_s(\mu)} + \frac{(33 - 2n_f)}{6\pi} \ln\left(\frac{Q}{\mu}\right)$$

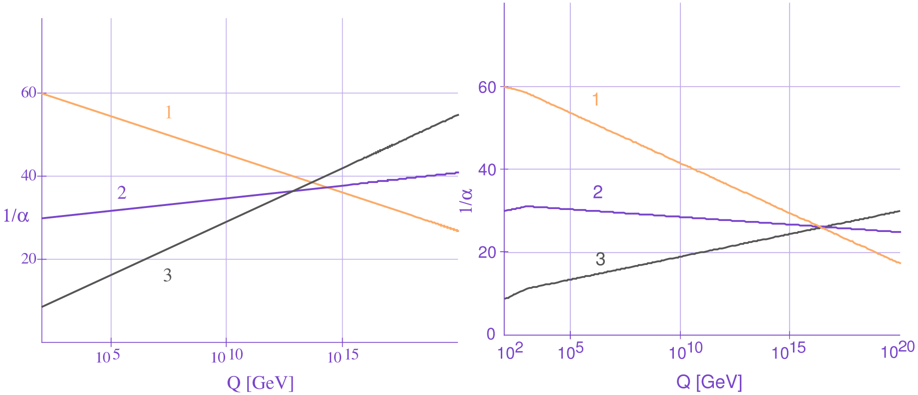
The spectrum matters! m_t influences low-energy α_s



$$\frac{1}{\alpha_s(2m_c)} \equiv \frac{27}{6\pi} \ln\left(\frac{2m_c}{\Lambda_{\mathrm{QCD}}}\right) \rightsquigarrow \Lambda_{\mathrm{QCD}} \propto m_t{}^{2/27}$$

Coupling constant evolution

SU(5) + TeV-scale SUSY



The agent of electroweak symmetry breaking represents
a novel fundamental interaction
at an energy of a few hundred GeV.

$\boxed{\textit{We do not know the nature of the new force.}}$

Inspired by the Meissner effect, we describe the EWSB interaction as an analogue of the Ginzburg–Landau picture of superconductivity.

light Higgs boson $\Leftrightarrow$ perturbative dynamics
heavy Higgs boson $\Leftrightarrow$ strong dynamics

What is the nature of the mysterious new force that hides electroweak symmetry?

- A fundamental force of a new character, based on interactions of an elementary scalar
- A new gauge force, perhaps acting on undiscovered constituents
- A residual force that emerges from strong dynamics among the weak gauge bosons
- An echo of extra spacetime dimensions

We have explored examples of all four, theoretically.

$\boxed{\text{Which path has Nature taken?}}$

Essential step toward understanding the new force that shapes our world:

Find the Higgs boson and explore its properties.

- Is it there? How many?
- Verify $J^{PC} = 0^{++}$
- Does H generate mass for gauge bosons, fermions?
- How does H interact with itself?

$\boxed{\textit{Finding the Higgs boson starts a new adventure!}}$

10 years precise measurements: no significant deviations

Quantum corrections tested at $\pm 10^{-3}$

No "new physics" ... yet!

Theory tested at distances from 10^{-17} cm to $\sim 10^{22}$ cm

origin Coulomb's law (tabletop experiments)

smaller $\left\{ \begin{array}{l} \text{Atomic physics} \rightarrow \text{QED} \\ \text{high-energy expts.} \rightarrow \text{EW theory} \end{array} \right.$

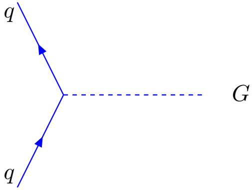
larger $M_\gamma \approx 0$ in planetary ... measurements

Is EW theory true? Is it complete ??

But what about gravity?

Natural to neglect gravity in particle physics ...

$$G_{\text{Newton}} \textit{ small } \iff M_{\text{Planck}} = \left(\frac{\hbar c}{G_{\text{Newton}}}\right)^{\frac{1}{2}} \approx 1.22 \times 10^{19}\ \text{GeV } \textit{large}$$



$$\sim \frac{E}{M_{\text{Planck}}}$$

$$\text{Estimate } B(K \to \pi G) \sim \left(\frac{M_K}{M_{\text{Planck}}}\right)^2 \sim 10^{-38}$$

300 years after Newton: Why *is* gravity weak?

But gravity is not always negligible ...

The vacuum energy problem

$$\text{Higgs potential } V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|\,(\varphi^\dagger\varphi)^2$$

At the minimum,

$$V(\langle\varphi^\dagger\varphi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda|\,v^4}{4} < 0.$$

$$\text{Identify } M_H^2 = -2\mu^2$$

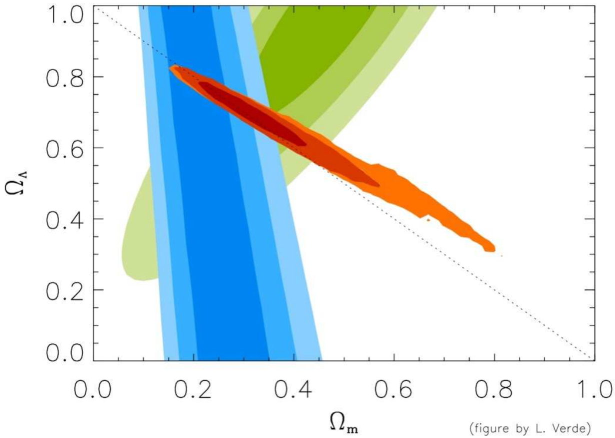
$V \neq 0$ contributes position-independent vacuum energy density

$$\boxed{\varrho_H \equiv \frac{M_H^2 v^2}{8} \geq 10^8\ \text{GeV}^4} \quad \approx 10^{24}\ \text{g cm}^{-3}$$

Adding vacuum energy density $\varrho_{\text{vac}} \Leftrightarrow$ adding cosmological constant Λ to Einstein's equation

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu} \qquad \Lambda = \frac{8\pi G_N}{c^4}\varrho_{\text{vac}}$$

Observed vacuum energy density $\varrho_{\text{vac}} \lesssim 10^{-46}\ \text{GeV}^4$



(figure by L. Verde)

$\varrho_H \gtrsim 10^8\ \text{GeV}^4$: mismatch by 10^{54}

A chronic dull headache for thirty years . . .

Parameters of the Standard Model

3	coupling parameters $\alpha_s, \alpha_{\text{EM}}, \sin^2\theta_W$
2	parameters of the Higgs potential
1	vacuum phase (QCD)
6	quark masses
3	quark mixing angles
1	CP-violating phase
3	charged-lepton masses
3	neutrino masses
3	leptonic mixing angles
1	leptonic CP-violating phase (+ Majorana . . .)
26^+	arbitrary parameters

parameter count not improved by unification

What the LHC *is not really* for . . .

- Find the Higgs boson,
  the Holy Grail of particle physics,
  the source of all mass in the Universe.
- Celebrate.
- Then particle physics will be over.

We are not ticking off items on a shopping list . . .

We are exploring a vast new terrain
. . . and reaching the Fermi scale

The Origins of Mass

(masses of nuclei "understood")

$p, [\pi], \rho$ understood: QCD

confinement energy is the source

"Mass without mass" Wilczek, *Physics Today* (Nov. 1999)

We understand the visible mass of the Universe

. . . without the Higgs mechanism

W, Z electroweak symmetry breaking

$$M_W^2 = \tfrac{1}{2}g^2v^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W$$
$$M_Z^2 = M_W^2/\cos^2\theta_W$$

$q, \ell^{\mp}$ EWSB + Yukawa couplings

ν_ℓ EWSB + Yukawa couplings; new physics?

All fermion masses $\Leftrightarrow$ physics beyond standard model

H ?? fifth force ??

Challenge: Understanding the Everyday

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?

Consider the effects of all the $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)}_Y$ *interactions!*

With no Higgs mechanism . . .

- Quarks and leptons would remain massless
- QCD would confine the quarks in color-singlet hadrons
- *N mass little changed,* but p outweighs n
- QCD breaks EW to EM, gives ($1/2500\times$observed) masses to W, Z, so weak-isospin force doesn't confine
- Rapid! β-decay $\Rightarrow$ lightest nucleus is n; no H atom
- Some light elements in BBN (?), but ∞ Bohr radius
- No atoms (as we know them) means no chemistry, no stable composite structures like solids and liquids

. . . the character of the physical world would be profoundly changed

Why the LHC is so exciting (II)

- Electroweak theory (unitarity) tells us the 1-TeV scale is special: Higgs boson or other new physics (strongly interacting gauge bosons)
- Hierarchy problem ⇒ other new physics nearby
- Our ignorance of EWSB obscures our view of other questions (e.g., identity problem). Lifting the veil at 1 TeV will change the face of physics

In a decade or two, we can hope to . . .

Understand electroweak symmetry breaking
Observe the Higgs boson
Measure neutrino masses and mixings
Establish Majorana neutrinos ($\beta\beta_{0\nu}$)
Thoroughly study CP violation in B decay
Exploit rare decays (K, D, . . .)
Observe n EDM, pursue e^- EDM
Use top as a tool
Observe new phases of matter
Understand hadron structure quantitatively
Uncover QCD's full implications
Observe proton decay
Understand the baryon excess
Catalogue matter & energy of universe
Measure dark energy equation of state
Search for new macroscopic forces
Determine GUT symmetry

Detect neutrinos from the universe
Learn how to quantize gravity
Learn why empty space is nearly weightless
Test the inflation hypothesis
Understand discrete symmetry violation
Resolve the hierarchy problem
Discover new gauge forces
Directly detect dark-matter particles
Explore extra spatial dimensions
Understand origin of large-scale structure
Observe gravitational radiation
Solve the strong CP problem
Learn whether supersymmetry is TeV-scale
Seek TeV dynamical symmetry breaking
Search for new strong dynamics
Explain the highest-energy cosmic rays
Formulate problem of identity

. . .

. . . learn the right questions to ask

. . . and rewrite the textbooks!

General References

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- "The Double Simplex," hep-ph/0509037.

Supplement: Higgs-boson Search

$$\Gamma(H \to f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$\propto M_H$ in the limit of large Higgs mass; $\propto \beta^3$ for scalar

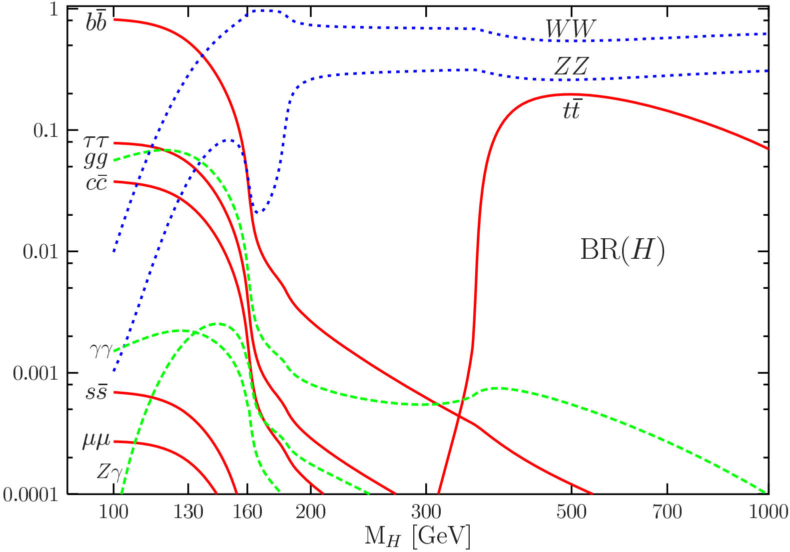
$$\Gamma(H \to W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}}(1-x)^{1/2}(4-4x+3x^2) \quad x \equiv 4M_W^2/M_H^2$$

$$\Gamma(H \to Z^0Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}}(1-x')^{1/2}(4-4x'+3x'^2) \quad x' \equiv 4M_Z^2/M_H^2$$

asymptotically $\propto M_H^3$ and $\frac{1}{2}M_H^3$, respectively

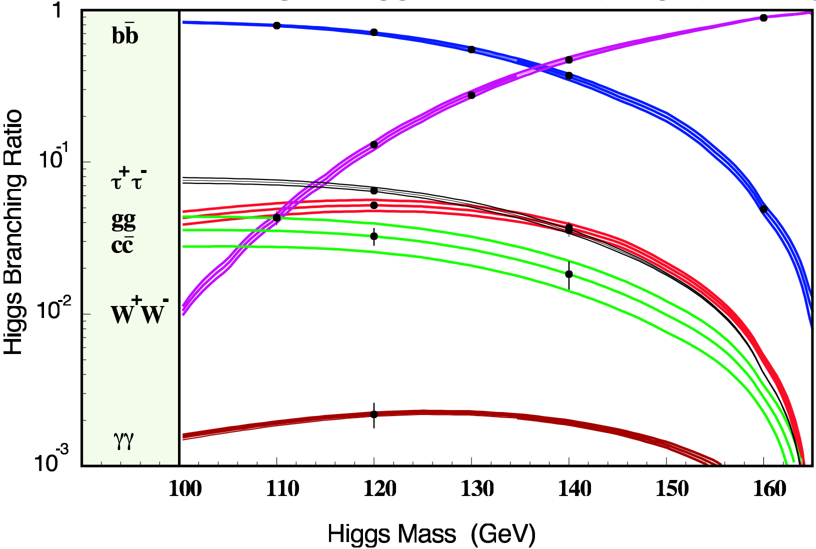
$2x^2$ and $2x'^2$ terms $\Leftrightarrow$ decays into transverse gauge bosons
Dominant decays for large M_H: pairs of longitudinal weak bosons

SM Higgs Boson Branching Fractions



Djouadi, hep-ph/0503172

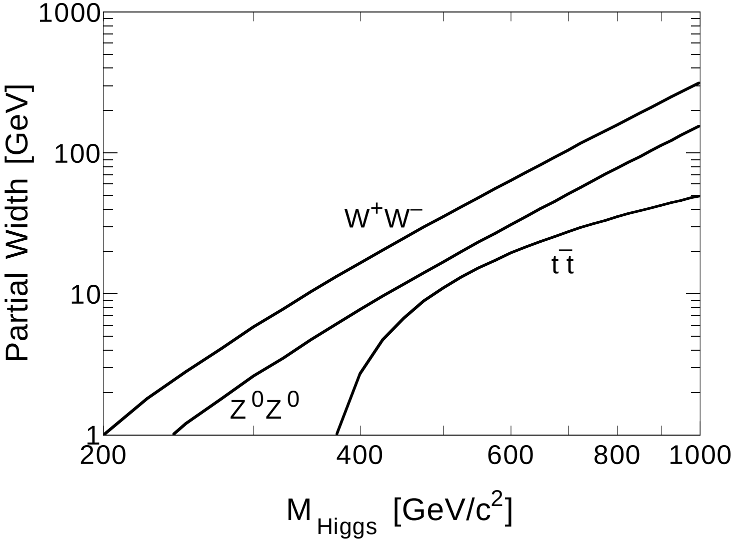
ILC would measure light Higgs-boson couplings precisely



Points: 500 fb^{-1} @ 350 GeV   Bands: theory uncertainty (m_b)

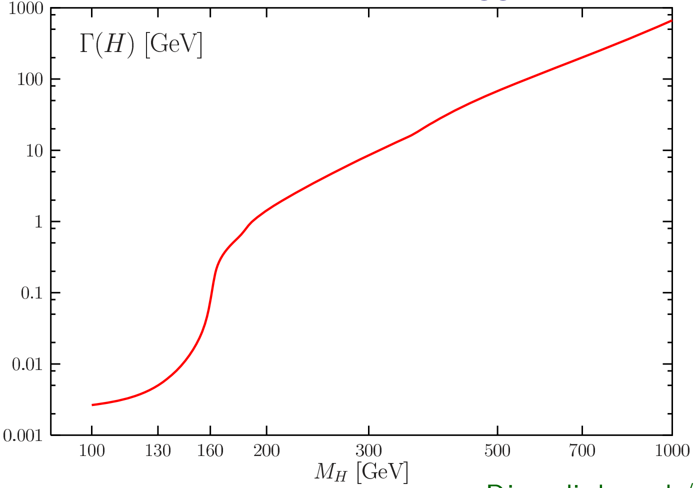
Battaglia +

Dominant decays at high mass



For $M_H \to 1$ TeV, Higgs boson is *ephemeral:* $\Gamma_H \to M_H$.

Total width of the standard-model Higgs boson



Djouadi, hep-ph/0503172

Below W^+W^- threshold, $\Gamma_H \lesssim 1$ GeV

Far above W^+W^- threshold, $\Gamma_H \propto M_H^3$

A few words on Higgs production ...

$e^+e^- \to H$: hopelessly small
$\mu^+\mu^- \to H$: scaled by $(m_\mu/m_e)^2 \approx 40\,000$
$e^+e^- \to HZ$: prime channel

Hadron colliders:
$gg \to H \to b\bar{b}$: background ?!
$gg \to H \to \gamma\gamma$: rate ?!
$\bar{p}p \to H(W, Z)$: prime Tevatron channel

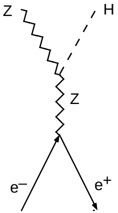
At the LHC:
Many channels accessible, search sensitive up to 1 TeV

Higgs search in e^+e^- collisions

$\sigma(e^+e^- \to H \to \text{all})$ is *minute*, $\propto m_e^2$

Even narrowness of low-mass H is not enough to make it visible . . . Sets aside a traditional strength of e^+e^- machines—*pole physics*

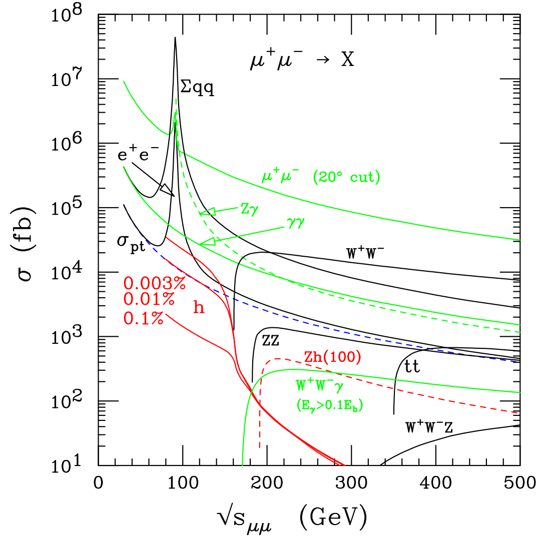
Most promising:
associated production $e^+e^- \to HZ$
(has no small couplings)



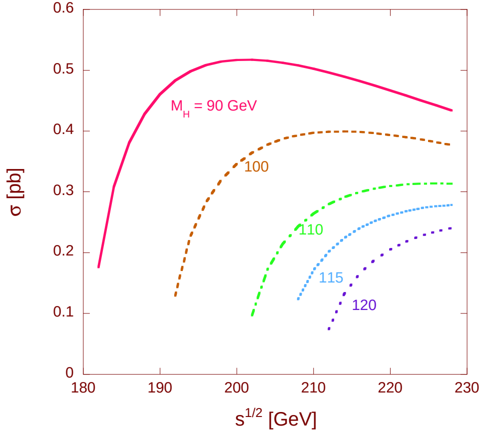
$$\boxed{\sigma = \frac{\pi\alpha^2}{24\sqrt{s}} \frac{K(K^2 + 3M_Z^2)[1 + (1 - 4x_W)^2]}{(s - M_Z^2)^2 \quad x_W^2(1 - x_W)^2}}$$

K: c.m. momentum of H $\qquad$ $x_W \equiv \sin^2\theta_W$

$\ell^+\ell^- \to X \ldots$



$$\sigma(e^+e^- \to H) = (m_e/m_\mu)^2 \sigma(\mu^+\mu^- \to H) \approx \sigma(\mu^+\mu^- \to H)/40\,000$$



+ important effect of ISR

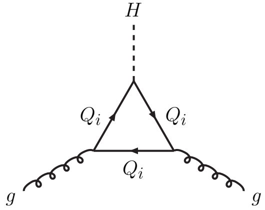
LEP 2: sensitive nearly to kinematical limit $\boxed{M_H^{\max} = \sqrt{s} - M_Z}$

LC: sensitive for $M_H \lesssim 0.7\sqrt{s}$

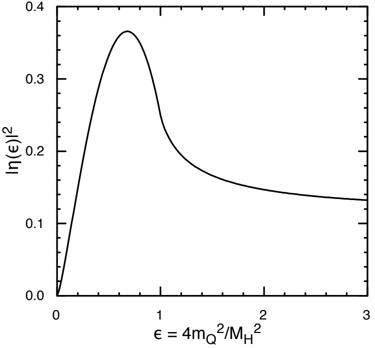
& measure excitation curve to determine

$$\delta M_H \approx 60\ \text{MeV}\ \sqrt{100\ \text{fb}^{-1}/\mathcal{L}}\ \text{for}\ M_H = 100\ \text{GeV}$$

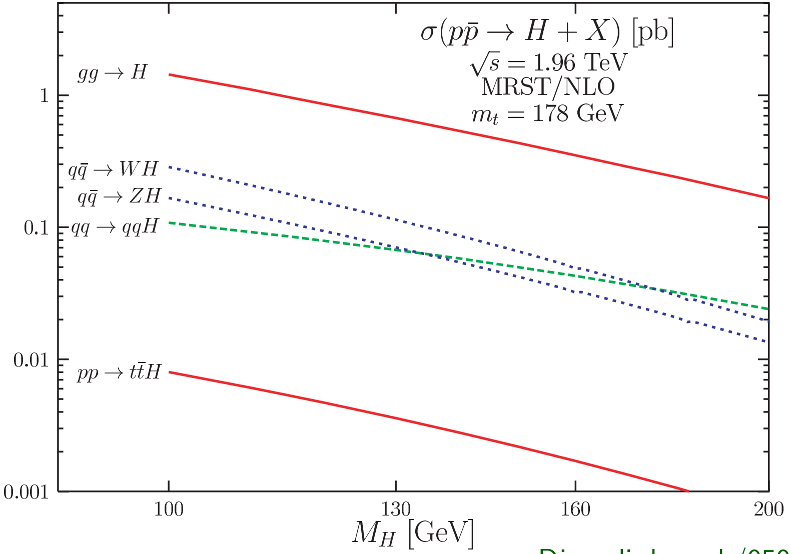
H couples to gluons through quark loops



Only heavy quarks matter:

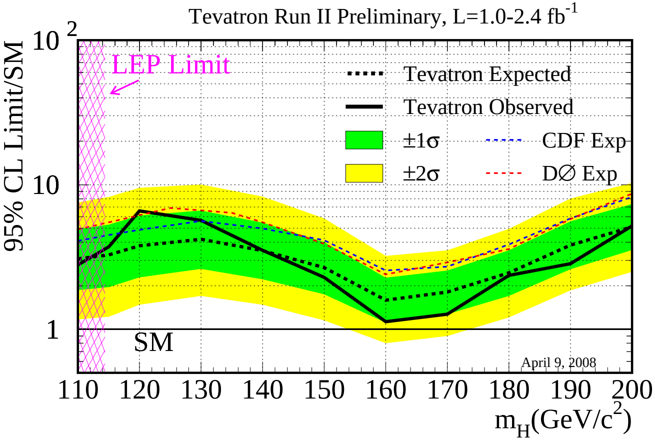


Higgs-boson production at the Tevatron



Djouadi, hep-ph/0503172

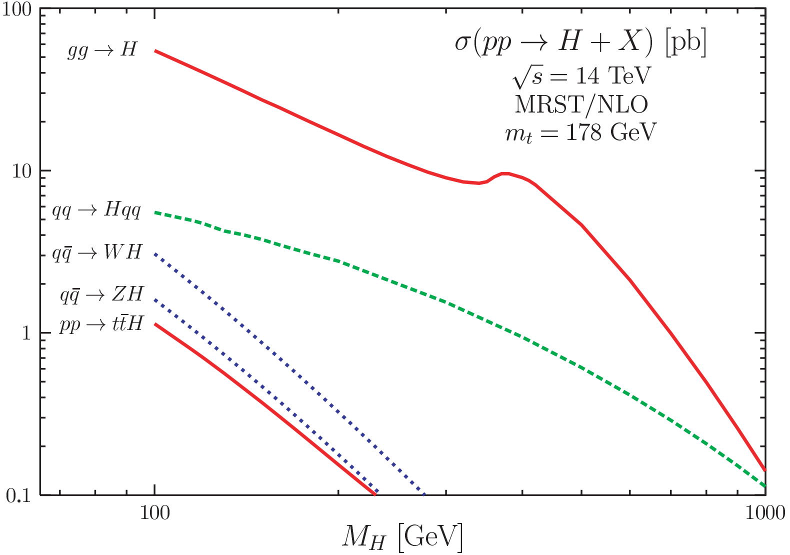
Current Tevatron Sensitivity



combining experiments, channels

Winter 2008

Higgs-boson production at the LHC



Djouadi, hep-ph/0503172

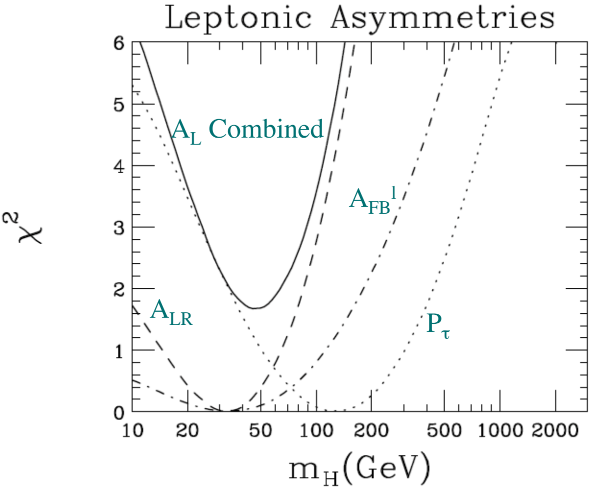
A Cautionary Note

- A_{FB}^{b}, which exerts the greatest "pull" on the global fit [slide 101], is most responsible for raising M_H above the range excluded by direct searches [slide 122].
- Leptonic and hadronic observables point to different best-fit values of M_H
- Many subtleties in experimental and theoretical analyses

M. Chanowitz, *Phys. Rev. Lett.* **87**, 231802 (2001); *Phys. Rev. D***66**, 073002 (2002); hep-ph/0304199;
http://phyweb.lbl.gov/∼chanowitz/rpm-10-06.pdf

Introduction to global analyses: J. L. Rosner, hep-ph/0108195; hep-ph/0206176

χ^2 Distributions: Leptonic Asymmetries



M. Chanowitz RPM 10/26/06 20

χ^2 Distributions: Hadronic Asymmetries